MATH TOURNAMENT 2012 CIPHERING – with ANSWERS

PART I

1. Find the value of the sum $1-2+3-4+\cdots+2011-2012$.

ANS: −1006

2. Find the integer k that satisfies both inequalities $\frac{k}{64} \le \frac{4}{5} \le \frac{k+1}{65}$.

ANS: 51

3. Find the largest perfect square less than 500, which has no digit greater than 3.

ANS: 121

4. Let x, y, and z be positive numbers such that $\frac{x^2}{x^2+1}+\frac{y^2}{y^2+1}+\frac{z^2}{z^2+1}=1$. Find the value of $\frac{1}{x^2+1}+\frac{1}{y^2+1}+\frac{1}{z^2+1}$.

ANS: 2

5. Suppose that $x^3-7x+k=0$ and $x^3-10x+2k=0$ have a common root in x. Find the possible value(s) of k.

ANS: 0,±6

6. Find the number of integer pairs (x, y) that satisfy the equation $xy=2x+2y$.

ANS: 6

7. Let $a>b>0$ such that $a^2+b^2=3ab$. Find the exact value of $\frac{a+b}{a-b}$.

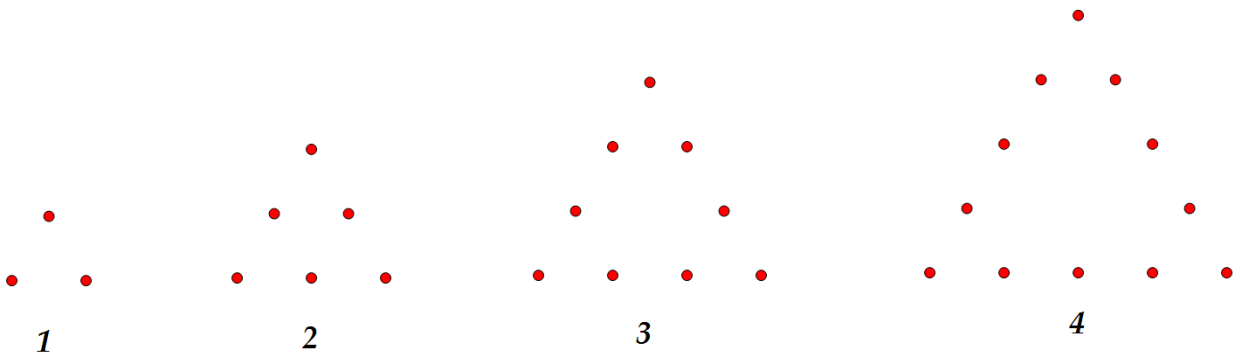
ANS: $\sqrt{5}$

8. In the experiment of rolling an 8-sided fair die, a regular octahedron with faces numbered from 1 to 8, determine the probability that the outcome is a prime number.

ANS: $\frac{1}{2}$

PART II

9. Based on the growing pattern of dots given below, how many dots are there in the 100th figure?



ANS: 300

10. Find the integer value of x that satisfies the equation $\sqrt{14+6\sqrt{5}}-\sqrt{14-6\sqrt{5}}=\sqrt{x}$.

ANS: 20

11. The entrance of a one-way tunnel is in the shape of a semi-ellipse of 16 ft base length and is just high enough for an 8 ft wide 9 ft tall truck to pass under. How high is the tunnel?

ANS: $\sqrt{108}$ **or** $6\sqrt{3}$ **ft**

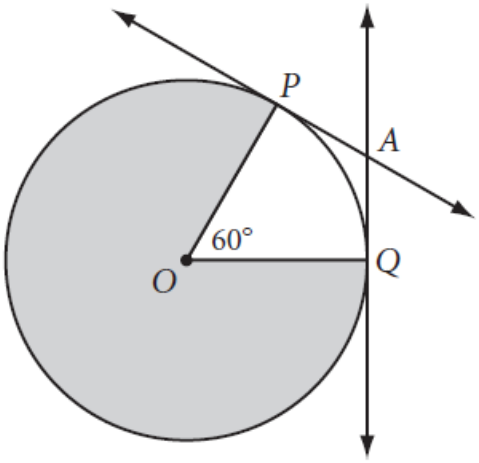
12. Given that $i^2 = -1$, evaluate the expression $\left(\frac{i+1}{i-1}\right)^2$.

ANS: –1

13. Determine the expression e^K where $K = \sum_{i=1}^{n} \ln(i)$.

ANS: $n!$

14. Lines $\overleftrightarrow{AP}$ and $\overleftrightarrow{AQ}$ are tangent to circle O, and $AP = 1$. Find the area of the shaded region.



ANS: $\frac{5\pi}{2}$

15. A sphere is inscribed in a cylinder with the same radius. Find the ratio of the volume of the sphere to the volume of the cylinder.

ANS: $\frac{2}{3}$

16. What is the geometric name of a quadrilateral with 4 congruent sides?

ANS: Rhombus