1. For how many positive integers $n \geq 1$ do both n^2 and n^3 have the same number of digits in base 10?

2. Find all solutions of the equation $2 \ln x = \ln 3 + \ln(x + 6)$

3. What digit is in the 2010th place in the decimal expansion of $\dfrac{4}{11}$?

4. In the triangle $\triangle ABC$ below we have $AB = AD = DC$ and the measure of angle $\angle(BAD)$ is 28°. What is the measure of the angle $\angle(ACD)$?
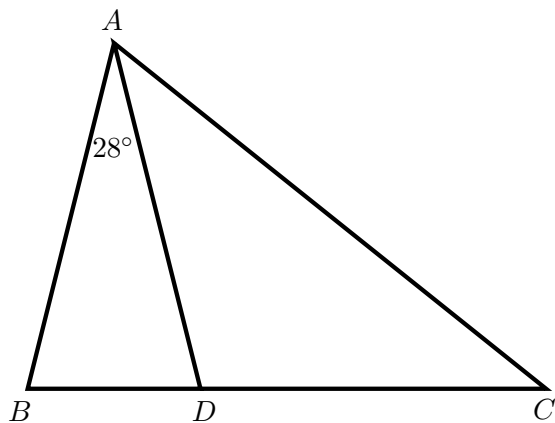
5. Find the sum $2010 + 4020 + 6030 + 8040 + \cdots + 201000$

6. If a, b, and c are real numbers not equal to 1 and such that

$$\frac{1}{a-1} + \frac{1}{b-1} + \frac{1}{c-1} = 1$$

find the value of

$$\frac{a}{a-1} + \frac{b}{b-1} + \frac{c}{c-1}$$

7. How many integers between 1 and 2010 are divisible by 67 and not by 3?

8. Because of a typo the pages of a book are numbered 1, 2, 3, 5, 6, 7, 9, ... (each 4th number is skipped). If the last page of the book is numbered 101 how many pages does the book have?

1. What is the next term in the infinite sequence $2, 3, 5, 9, 17, \ldots$?

2. Solve the equation $\sqrt{2x - 1} + 2 = x$

3. If $\sin(\alpha) = \dfrac{2}{3}$ then what is the numerical value of $\cos(2\alpha)$?

4. Out of 400 fish that swim in a water tank 1% are blue and the rest are yellow. How many yellow fish should be removed from the tank such that the blue fish represent 2% of the remaining fish?

5. What is the largest possible area of a right triangle inscribed in a circle of radius 2010?

6. How many positive integers are in the set

$$A = \{\sqrt{1}, \sqrt{2}, \sqrt{3}, \ldots, \sqrt{2009}, \sqrt{2010}\}?$$

7. There are 21 boys and 37 girls in a dance band. Each week 6 new boys and 4 new girls join the band until the number of boys and girls in the band is the same. How large is the band when this happens?

8. The sum of the first 4 terms of a geometric sequence with common ratio of 2 is 2010. What is the 1st term?

Round 1 Key

1	2	3	4	5	6	7	8
3	$x = 6$	6	38°	10150500	4	20	76

Round 2 Key

1	2	3	4	5	6	7	8
33	$x = 5$	$\dfrac{1}{9}$	200	$2010^2 = 4040100$	44	138	134