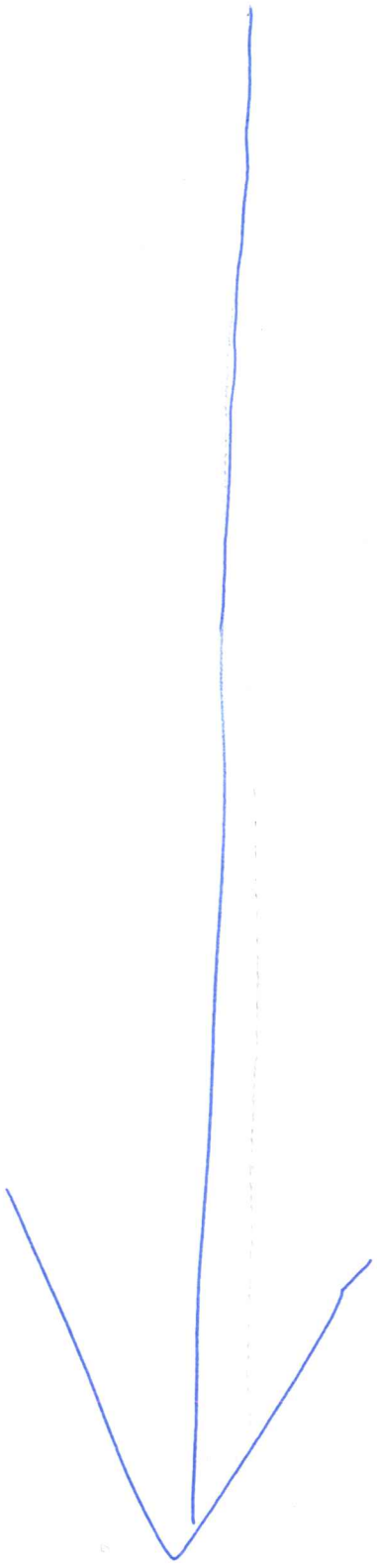


Problem 21

- **Let $a_0 = 1$, and $a_{n+1} = a_n + e^{-a_n}$ for $n \geq 0$. What is the limit of $\frac{e^{a_n}}{n}$?**

(A) 0 (B) 1 (C) 1/2 (D) 2 (E) 2/3 (F) ∞

Problem 22

- The real-valued continuously differentiable function $f(x)$ has the property

$$2f(x)^3 = \int_0^x [f(t)^4 + 9f'(t)^2]dt + \frac{1}{4 \cdot 337^3}$$

for all $x < 2022$. Find $f(3)^{-1}$.

(A) 670 (B) 671 (C) 673 (D) 674 (E) 675 (F) 679

Problem 23

- **Find the number of solutions (a, b), of the Diophantine equation**

$$\frac{1}{a} + \frac{1}{b} = \frac{5}{2022}$$

where a and b are positive integers and $a < b$.

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4 (F) 5

Problem 24

- We let n be the smallest natural number with the property that for every integer m such that $0 < m < 2022$, there exits an integer k such that

$$\frac{m}{2022} < \frac{k}{n} < \frac{m+1}{2023}.$$

What is the sum of the digits of n?

(A) 8 (B) 9 (C) 10 (D) 11 (E) 12 (F) 13

Problem 25

- Two real values x and y are chosen at random from the interval $(0,1)$ (with uniform distribution). The probability that the closest integer to $\frac{x}{y}$ is odd is equal to $\frac{\pi+m}{n}$ for some integers m and n. What is $n-m$?

(A) 5 (B) 6 (C) 7 (D) 8 (E) 9 (F) 10

Problem 26

- Consider the real valued function $f(x)$ which is twice differentiable on $\mathbb{R}$ and satisfies the conditions

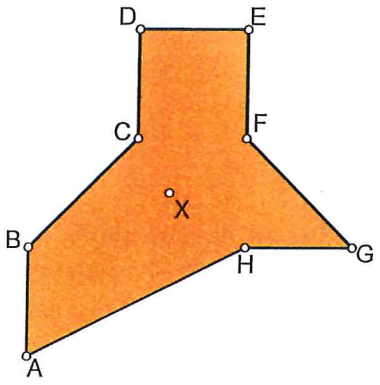
$$f''(x) + (x^3 - x^2 + x)f'(x) + f(x) = 0$$

for all x, $f(0) = 2$ and $f'(0) = 0$. What is the maximum of f on $\mathbb{R}$?

(A) 1 (B) 2 (C) 3 (D) 4 (E) 5 (F) 6

Problem 27

- In the accompaning figure, the octogon $ABCDEFGH$ is a lattice polygon, i.e., all vertices have integer coordinates which are $A(0,0)$, $B(0,1)$, $C(1,2)$, $D(1,3)$, $E(2,3)$, $F(2,2)$, $G(3,1)$ and $H(2,1)$. The center of mass is X and its coordinates are (x_G, y_G), with $x_G = \frac{m}{n}$ a rational written in reduced form. What is $m-n$?



(A) 10 (B) 9 (C) 7

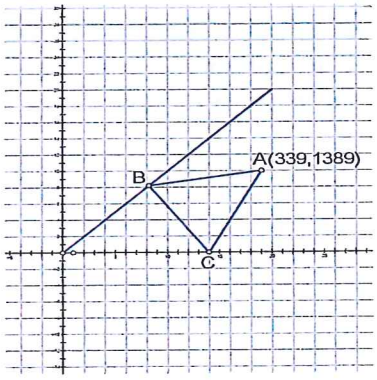
(D) 5 (E) 12 (F) 11

Problem 28

- Consider the point A of coordinates $(339, 1389)$ and two more points, B on the line of equation $y = x$ and C on the x-axis. What is the minimum perimeter of such a triangle ABC?

(A) 2022 (B) $2022 \cdot 2$ (C) $\frac{2022}{2}$

(D) $\frac{2022}{3}$ (E) $\frac{2022}{6}$ (F) $2022 \cdot 3$

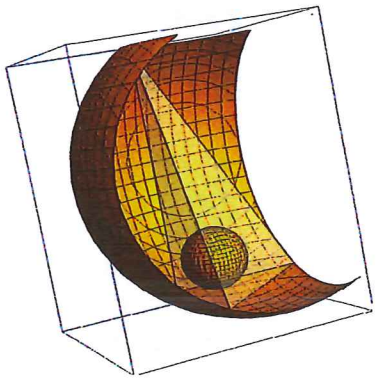


Problem 29

- In the accompaning figure, we have a tetrahedron inscribed in a shpere of radius R and circumscribed to a sphere of radius r. Knowing that the coordinates of the tetrahedron's vertices are $(0,0,0)$, $(1,0,0)$, $(0,1,0)$ and $(0,0,2)$, what is R^2/r^2 ?

(A) 36 (B) 20 (C) 16

(D) 18 (E) 24 (F) 9



Problem 30

- In the adjacent figure, there are identical (rectangular) bricks of dimension 1×2 stacked in such a way they are barely in equilibrium (only gravitation acts on them and the one on the bottom is perfectly horizontal) and the vertical projection of the top brick is as far as possible from the bottom brick. What is the distance LM?

(A) 1/12 (B) 1/6 (C) 1/7

(D) 2/7 (E) 1/8 (F) 1/9