Problem 11

- The sequence of natural numbers $\{x_n\}$ is defined in the following way: $x_1 = 1$, $x_2 = 7$ and for every $n \geq 3$, we choose x_n to be the smallest natural number k which is not in the set

$$\{x_1, x_2, ..., x_{n-1}\}$$

and it is relatively prime with x_{n-1}, i.e., $\gcd(k, x_{n-1}) = 1$. For example $x_3 = 2$ since 2 is not in $\{1, 7\}$ and it is the first number relatively prime with 7. The sequence starts like

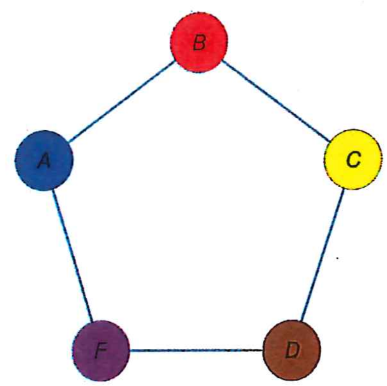
$$\{1, 7, 2, 3, 4, 5, 6, 11, 8, 9\}.$$

What is the term x_{212}?

(A) 210 (B) 221 (C) 212 (D) 215 (E) 211 (F) 217

Problem 12

- The regular pentagon $ABCDF$ is *properly colored* if every side has endpoints colored differently (see an example of such a proper coloring in the accompanining figure–coloring A and B with the same color is not allowed, but coloring A and C with the same color is fine). The number of proper colorings of the labeled graph in the accompanining figure using 5 colors is equal to N, and then $2022 = 2N - x$. What is x?

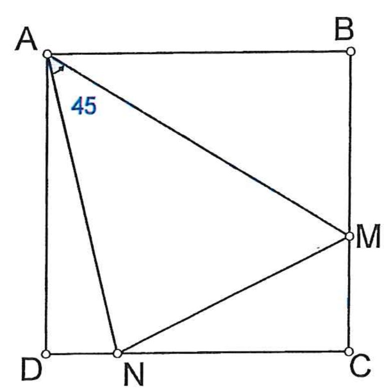


(A) 15 (B) 16 (C) 17

(D) 18 (E) 19 (F) 20

Problem 13

- Let $ABCD$ be a unit square. The points M and N lie on the sides BC and CD respectively such that $\angle MAN = 45°$. If $MN = \frac{5}{6}$, what is $\frac{1}{BM \cdot DN}$?



(A) 1 (B) 2 (C) 3

(D) 4 (E) 5 (F) 6

Problem 14

- The quadratic polynomial $f(x) = x^2 + bx + c$ has the property that the polynomial $f(f(x)) - x$ has roots $\pm\frac{2}{3}$ but none of these are roots of $f(x) - x$. If the roots of $f(x)$ are x_1 and x_2, what is $3|x_1 - x_2|$?

(A) 1 (B) 2 (C) 3 (D) 4 (E) 5 (F) 6

Problem 15

- We let $z = \cos\frac{2\pi}{7} + i\sin\frac{2\pi}{7}$ (i is the pure imaginary number). The value of $z + z^2 + z^4$ can be simplified to $\frac{m+i\sqrt{n}}{2}$ for some integers m and n. What is $m + n$?

(A) 1 (B) 2 (C) 3 (D) 4 (E) 5 (F) 6

Problem 16

- The natural number N has positive (integers) divisors $1 = d_1 < d_2 < \cdots < d_k = N$. Knowing that $N = d_{20}d_{22}$ and N is even, what is $N \pmod 7$?

(A) 1 (B) 2 (C) 3 (D) 4 (E) 5 (F) 6

Problem 17

- The surface in 3D of equation $x^3 + y^3 + z^3 - 3xyz = 27$ intersected with the plane $x + y + z = 18$ is a circle of radius r. What is r?

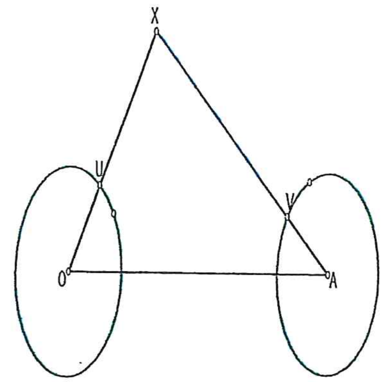
(A) 1 (B) 2 (C) 3 (D) 4 (E) 5 (F) 6

Problem 18

- Consider the unit segment $\overline{OA}$, and two small segments $\overline{OU}$ and $\overline{AV}$ which spin freely and independently, around O and A respectively. They stop at random positions and we denote by X the intersection of the rays $\overline{OU}$ and $\overline{AV}$. What is the probability that $XO < 1$?

(A) 1/12 (B) 1/6 (C) 1/4

(D) 1/3 (E) 2/3 (F) 1/2



Problem 19

- **The number N multiplied by 2021 contains the number 2022 as one of its consecutive digits. For example 10005 multiplied by 2021 is 20220105 which contains 2022 as its first four digits. Knowing that N is the smallest such natural number, what is the sum of its digits?**

(A) 11 (B) 17 (C) 7 (D) 20 (E) 15 (F) 13

Problem 20

- The real numbers x, y, and z satisfy

$$2021x + 2022y + 2023z = 0 \quad \text{and} \quad x^2 + y^2 + z^2 = 2xy + 2yz + 2zx.$$

What is the minimum value of $x + y + z$?

(A) 0 (B) -3 (C) -1 (D) 2 (E) -4 (F) -5