Painted Cubes

Part 1

Your little brother dropped your Rubik's Cube in a bucket of purple paint. Rats! Now all the faces are painted the same color. For these questions, assume that your cube is solid. How many of the individual cubes in the 3 by 3 by 3-cube have 0 faces painted purple? How many have 1 face painted purple? How many have 2 faces painted purple? How many have 3 faces painted purple?

Organize your answer so that it is easy to read and makes sense.

What if you had a 4 by 4 by 4-cube and the same thing happened? Record the number of faces painted purple for each individual cube in the bigger cube in the same way that you did for the 3 by 3 by 3.

Now determine the same information for a 5 by 5 by 5-cube.

When you have completed this task, take your organized, recorded data to Room 213 to present to the Master Teacher. Be prepared to articulate your solution to the Master Teacher.

Part 2

Write a formula for an n by n by n cube (for $n \geq 2$) that will produce the correct number of individual cubes with 0 faces painted, 1 face painted, 2 faces painted, and 3 faces painted.

When you have a formula and can explain it, go to Room 213 to present your solution to the Master Teacher.