

PUZZLE



Class Average



In Professor Cody's mathematics class, 36 students took the final exam. If the average passing grade was 78, the average failing grade was 60, and the class average was 71, how many of these 36 students passed the final?



SOLUTION

Solution: 22

If 36 students took the exam, the average passing grade was 78, the average failing grade was 60, and the class average was 71 then:

$$\frac{78(x) + 60(36 - x)}{36} = 71 \text{ and } x = 22$$



PUZZLE



Who Needs Change?

Archimedes, Brahmagupta, Carl, and Diophantus has just finished lunch in a restaurant and were paying their checks.

- ❖ The four men, each of whose coins totaled one dollar, had the same number of silver coins.
- ❖ Archimedes had exactly three quarters, Brahmagupta had exactly two quarters, Carl had exactly one quarter, and Diophantus had no quarters.
- ❖ The four men had to pay the same amount; three paid the exact amount, but the fourth required change.

Who required change?

Note: "Silver coins" may be nickels, dimes, quarters, or half dollars.

SOLUTION



Who Needs Change?

Solution:

From Clue 2, Archimedes had three quarters. So, then from Clue 1, Archimedes had either (Q represents quarter, D represents dime, and N represents nickel):

$QQQDDN$, $QQQDNNN$, or $QQQNNNNN$

Then, from clue 1, the number of coins each man had was either six, seven, or eight. Trial and error (Trial and revise) reveals that a combination of six coins with two quarters and a combination of eight coins with one quarter are impossible. So, the number of coins is seven. The different combinations are shown below (*H* represents half dollar).

Six Coins	Seven Coins	Eight Coins
QQQDDN	QQQDNNN	QQQNNNNN
QQ????	QQDDDDD	QQDDDDNN
QHDNNN	QHNNNNN	Q???????
HDDDDD	HDDDDNN	HDDDNNNN

Then, from Clue 3, the amount of cents of each check was one of the following: 5, 10, 15, 20, 25, 30, 35, 40, 45, 50, 55, 60, 65, 70, 75, 80, 85, 90, 95, 100. Assuming the amount of each check was each amount in turn, one discovers that all four men could have paid the exact amount in all cases except when the amount in cents was 5, 15, 85, or 95. If the amount in cents was 5, 15, 85, or 95, only the man with two quarters, Brahmagupta, could NOT have paid exactly. **So, Brahmagupta required change.**

PUZZLE

All Smiles Part One

This is a two-part problem.



Stage 1



Stage 2



Stage 3

Part One:

Without algebra, describe how you see the shapes growing. Once you have one description, look again to find another way. Report both of your descriptions to the Master Teacher. When the Master Teacher is satisfied with your descriptions, you will receive part two of this problem.

Your task is to create your descriptions.

Be prepared to justify your descriptions.

Problem Scoring

Part One
1 point

All Smiles: Part Two

Part Two:

What does Stage 4 look like? How many smiling faces are in Stage 4? Generate a formula that will determine the total number of smiling faces necessary to build the n^{th} case for the growing shapes. Be prepared to justify each aspect of your formula with respect to the growth of the shapes or the structure of the geometric aspects of the shapes. You will earn 1 point for the drawing and the formula and an additional 2 points for the explanation.

Your task is to draw Stage 4 and derive the n^{th} formula.

Be prepared to justify your solution

Problem Scoring

Part Two

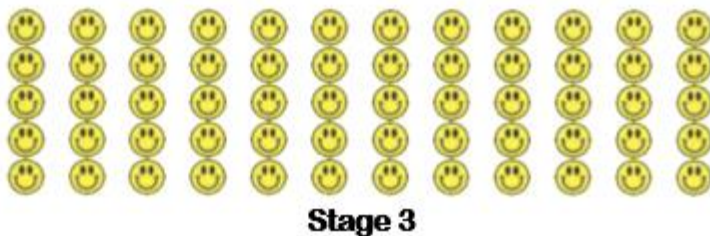
1 point for the drawing

2 points for the explanation

SOLUTION

All Smiles Part One

This is a two-part problem.



Part One:

Solution: answers will vary, stage 4 will have 96 smiling faces

SOLUTION

All Smiles: Part Two

Part Two:

Your task is to draw Stage 4 and derive the nth formula.

Be prepared to justify your solution

Solution $4n^2 + 8n$ 96 smiles

Student explanation