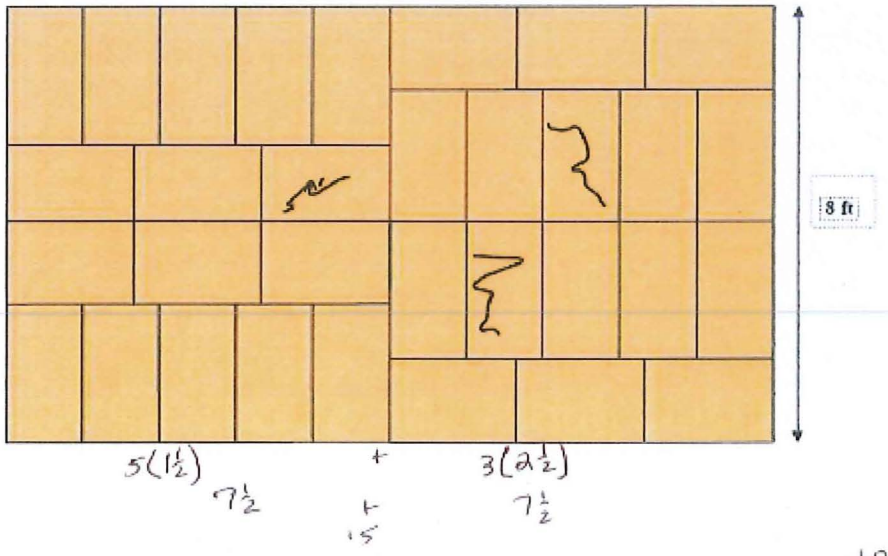
Mystery Patio Pavers

You work for a company that makes custom paver tiles. A customer sent you this picture of his patio. He said the patio is made up of identical tiles, positioned either vertically or horizontally. He wants to replace three tiles that are cracked. He didn't give you the dimensions of the tiles but did tell you that the width of the patio is 8 feet.



- What are the dimensions of a replacement tile for the patio? $1\frac{1}{2}$ ft x $2\frac{1}{2}$ ft
- What is the area of the patio? 8 x 15 = 120 sq ft.

Be prepared to justify your descriptions.

When you have solved the problem, go to Historic Room 1019 to present your solution to the Master Teacher.

Digital Equation

Using the digits 1 – 9 at most one time, fill in the boxes below to form the largest possible integer value for the sum of the values of *x* and *y*. A team may return to present this problem to receive the most points possible. The maximum integer solution is 3 points. An integer solution is worth 1 point.

$$2x - \square = \square$$

$$\square y + \square = \square$$

Largest integer sum is 14

$2x-6=8$ or $2x-8=6$
$1y+2=9$

or $2x-9=7$ and $1y+2=8$

Be prepared to justify your descriptions.

When you have solved the problem, go to Historic Room 1016 to present your solution to the Master Teacher.

number of triangles solution

11%

Strategy: Count in an organized way.

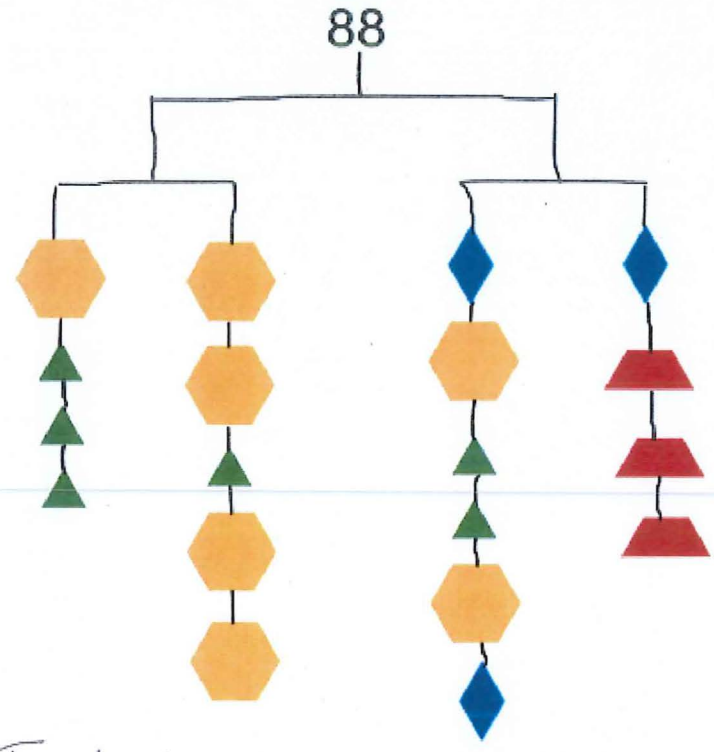
Label each single region separately. Then count the triangles which enclose only 1 region, then those that enclose 2 regions, and so on. Thus:

# of Regions	List of triangles by named regions	Subtotals
1	A, B, C, F, R, K, J, H, G, D	10 triangles
2	AB, BC, CF, FR, RK, KJ, JH, HG, GD, DA	10 triangles
3	ABC, CFR, RKJ, JHG, GDA; DEF, BEK, FEH, KED, HEB	10 triangles
5	BEHKJ, FEDHG, KEBDA, HEBFC, DEFKR	5 triangles

There are 35 triangles of all sizes in the diagram.

Pattern Block Mobile

The total value of all of the pieces in this mobile is 88. The Mobile is balanced as shown. What is the value of each piece? (You have a set of pattern blocks that you can use to help solve the problem.)



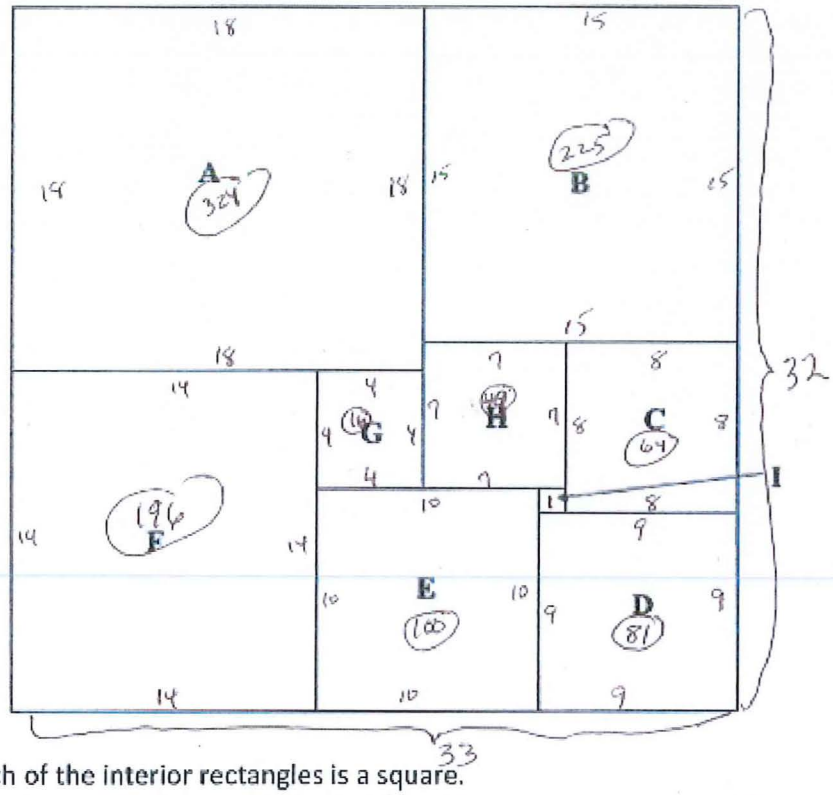
Triangle = 6 Hexagon = 4 Rhombus = 1 Trapezoid = 7

Be prepared to justify your descriptions.

When you have solved the problem, go to Historic Room 1015 to present your solution to the Master Teacher.

Squares, Squares, and More Squares

In November 1958, the magazine *Scientific American* showed this diagram on its cover.



- Each of the interior rectangles is a square.
- If square D is 81 square units and square C is 64 square units, what is the area of the entire figure? 1056 sq. units
- What is the perimeter of the outer figure? $2 \cdot 32 + 2 \cdot 33 = 130$

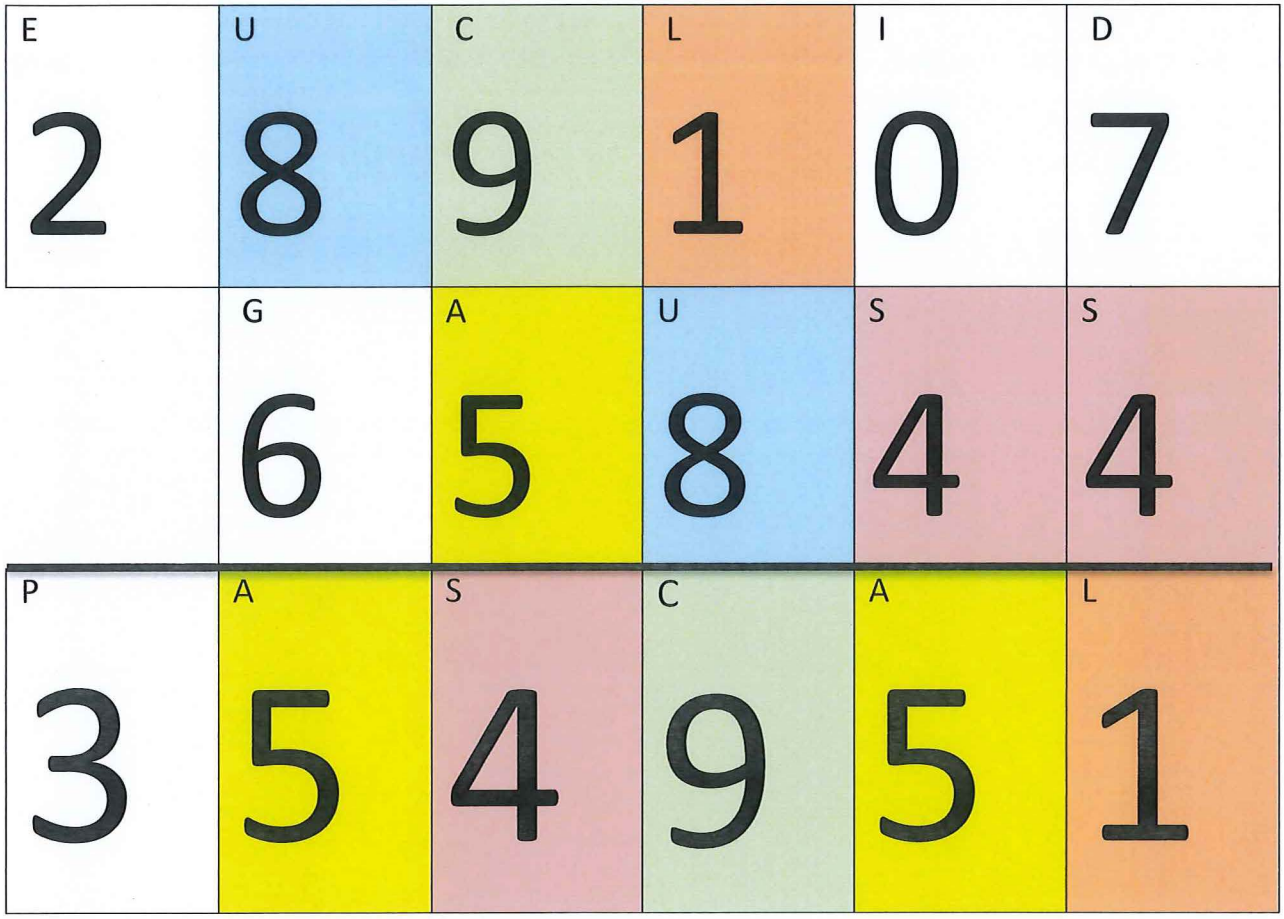
Be prepared to justify your descriptions.

When you have solved the problem, go to Historic Room 1056 to present your solution to the Master Teacher.

Area = 324 + 225 + 196 + 100 + 81 + 64 + 49 + 16 + 1 = 1056

Finding mathematical harmony situation

$$\text{Percentage increase} = \frac{0.7(2x) - x}{x} = 0.4 = 40\%$$


E
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1

Part I- Maximizing Cost Savings

Calculations for E85 $2.35 vs. Unleaded $3.35 (1 pt)

Cost per mile (this is what really matters)

To compare fairly, calculate **cost per mile**:

Regular

$$\frac{3.35\ \$/\text{gallon}}{13.6\ \text{miles/gallon}} \approx 0.246\ \text{dollars per mile}$$

= 24.6 cents per mile

E85

$$\frac{2.35\ \$/\text{gallon}}{12.3\ \text{miles/gallon}} \approx .191\ \text{dollars/mile}$$

= 19.1 cents per mile

Compare

E85 costs about:

24.6 cents per mile −19.1 cents per mile = 5.5 cents less per mile.

That's a **big difference** over time.

Final Answer

E85 saves you more money, despite lower MPG.

The price difference ($1.00 per gallon cheaper) more than offsets the 1.3 mpg loss in fuel economy.

You would need regular fuel to be around **$2.60 per gallon** to break even with E85 at $2.35.

Calculations for E85 $2.99 vs. Unleaded $3.17

Cost per mile (most important comparison)

Regular

$$\frac{3.17\ \$/\text{gallon}}{13.6\ \text{miles/gallon}} \approx .233\ \text{dollars/mile}$$

≈ 23.3 cents per mile

E85

$$\frac{2.99\ \$/\text{gallon}}{12.3\ \text{miles/gallon}} \approx .243\ \text{dollars/mile}$$

≈ 24.3 cents per mile

Compare

Regular is cheaper by:

24.3 cents per mile −23.3 cents per mile =1.0 cent per mile

So now **regular unleaded saves about 1 cent per mile**.

Final Conclusion

With these new prices:

Regular unleaded is now the better deal.

Because the price gap ($0.18 per gallon) is too small to offset the 1.3 mpg advantage regular has.

Part II - Maximizing Cost Savings

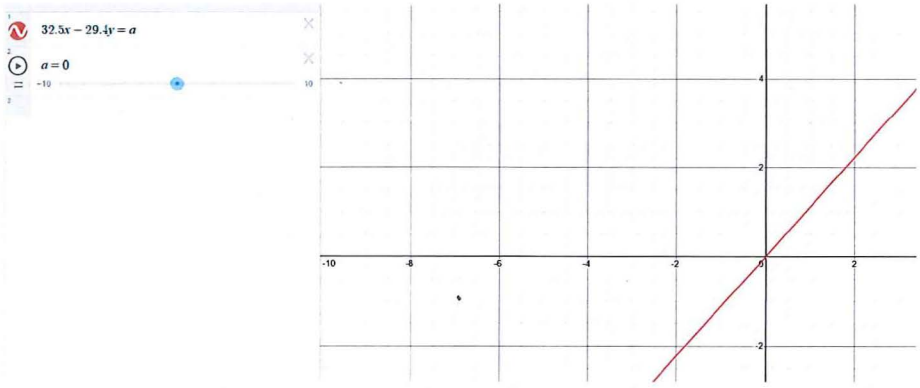
The truck uses approximately 1.3 mpg less when running regular unleaded fuel. This is approximately 12.3/13.6 = 90% as much fuel. Thus the threshold for cost savings at the pump is also when there is 90% more or less in dollars per gallon.

That is, when does x/y=90%? If the value for x/y is > 90%, then we are spending too much money on x, E85. We should purchase unleaded gasoline to save on the $/mile.

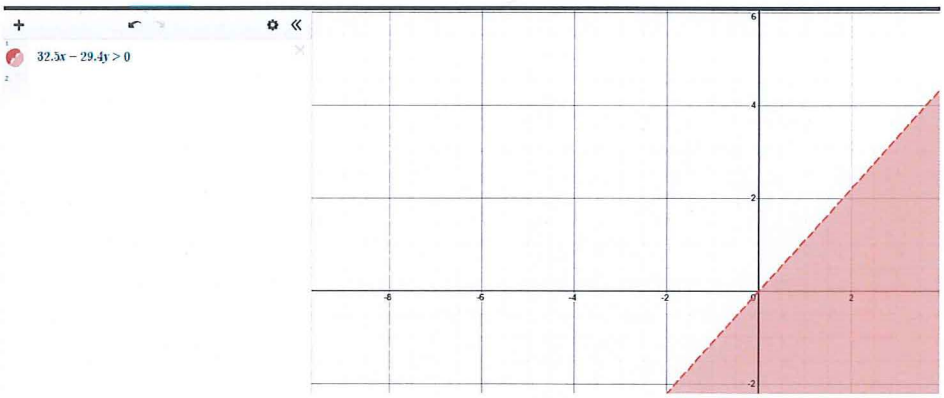
If x/y < 90%, the purchasing E85 fuel will save us in $/mile because we are under the 90% threshold of mile/gallon. This x and y rate is the slope in the below function explanation.

I thought about the problem by keeping the distance traveled and/or the gas used between the two fuel types consistent. The amount of fuel I use is really based on how far I go mathematically, so it doesn't have a variation according to the cost at the pump. I thought I needed to do this so I could just be changing one factor in the problem, the fuel cost. I also thought it made a lot of sense to do this because I always fill my tank up when it's empty and I don't put a limit on how far I drive just because I use a certain fuel type. Either way, I chose to use a distance of 400 miles for consistency. If I drove 400 miles with ethanol, this would be 400/12.3; about 32.5 gallons of gasoline. With regular fuel, I'd use about 400/13.6; about 29.4 gallons. So I will always use less fuel no matter the distance I travel with regular gas, but is it cheap enough to benefit me? **To calculate the cost, I thought I should multiple the cost per gallon times how much gas I buy. For ethanol this would be 32.5 gallons (times) gas cost for ethanol x. For regular this would be 29.4 gallons (times) the price of regular gas per gallon, say y.** So I have two equals: total cost of using ethanol 32.5x and total cost of using regular 29.4y**. So to determine which would be the best buy it matters whether the function 32.5x - 29.4y is positive or negative.**

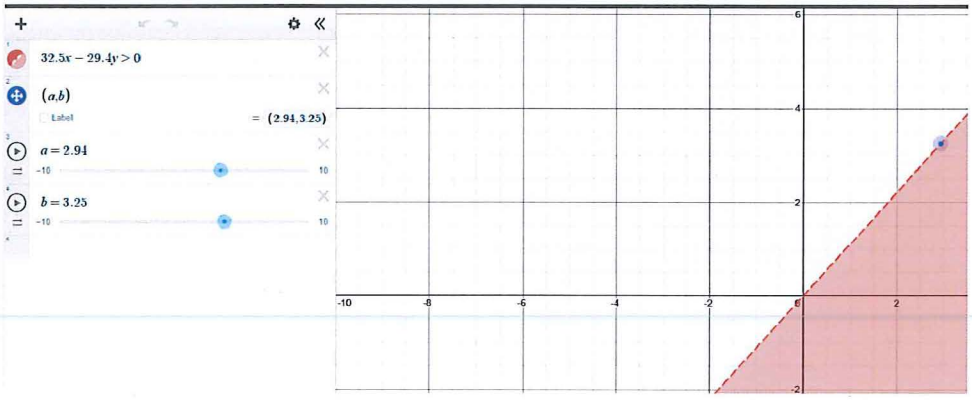
So I made a function to see this and put a slider for what it equals: https://www.desmos.com/calculator/mxmtz3ttpg.



But I noticed that this just moved my line up and down and the slope didn't change. Really, I was interested in when this graph or function was positive or negative: https://www.desmos.com/calculator/uf9ce9ylzi



Along this dotted line on the graph was when there was no difference between regular and ethanol: https://www.desmos.com/calculator/6txu3f9ofw.



For example, below $3.25/gal for regular and $2.94 for ethanol. If I go just left or just up from that point with my slider a, then I am either descreasing the price of ethanol or increasing the price of regular gas. Both of these would make the function positive and better to buy ethanol. Similarly, if I go right or down I will increase the price of ethanol or decrease the price of regular gas, bot a reason to buy regular gas. So the answer to my problem lies in the slope of this graph and whether a point is above or below this function or this slope. The slope appears to be about 1.1. So when I divide the gas prices at the pump and they are above 1.1, I should buy ethanol. If I divide the gas prices and they are less than 1.1, I should buy regular.

For instance, 3.25 for regular and 3.1 for ethanol would be 3.25/3.1 is 1.04; buy regular.

3.25/2.75 is 1.18, buy ethanol.

Important, I had a lot of constraints on this problem that fit my F250. I have a F150 also that has more fluctuation in fuel efficiency. I think I'll try that truck next time!

For a staircase of height one, it takes 3 blocks. For a staircase of height two, it takes the 3 previous blocks and an additional 9. As the pattern continues, we can create a table of values to assist us.

Staircase Height	# of cubes
1	3
2	12
3	27
4	58
...	...
100	????

One Solution Pathway...

Each new staircase adds a new slice or cross-section of cubes that is a rectangle. That rectangle always has a height of 3. The width of this rectangle changes at each stage. Consider the table below as a description for each layer.

Staircase Height	Height of rect. Layer	Width of rect. Layer
1	3	1
2	3	3
3	3	5
4	3	7
...	...	
x	3	2x-1
100	3	199

Thus, the base layer will have a rectangle that is 199x3. The next layer will have a layer that is 197*3. This pattern will continue. The total number of cubes would be the sum of each layer. That is

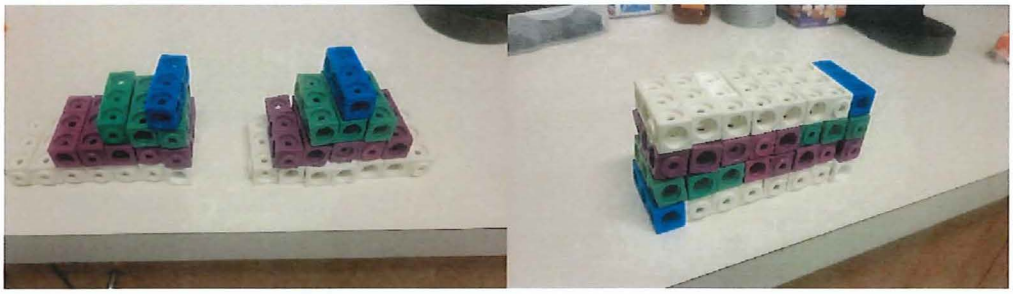
$$199*3 + 197*3 + 195*3 + 193*3 .... +7*3+5*3 +3*3 +1*3$$

This is the same as $3* (199 + 197 + 195 + 193 + ... 7 + 5 + 3 + 1)$.

Using the commutative property, we notice that 199 + 1 is 200; 197 + 3 is 200; 195 + 5 is 200; 193 + 7 is 200; and so on. We can find the sum of the odd numbers from 199 to 1 as 50 sets of 200. That sum would be 10,000. So 3*10,000 is 30,000.

Alternative Solution Pathway.....

Use the blocks to create two of the same staircases to see a pattern. When you slide the staircases over to one side, the staircases easily fit together to make a rectangular prism. This rectangular prism's volume is always 3x(staircase height)x(2xstaircase height). However, the volume of this prism is twice too much because we made two staircases, so the amount of cubes for any staircase like this can be found by 3x(staircase height)x(2xstaircase height)/2.



3 x 100 x 2 x 100 / 2 = 30,000.

Umbrella Radius Problem Solution:
The first step is to determine the hypotenuse of the umbrella. (the fabric radius length)
Write the number 4 ½ squared + 12 squared = H2
12.816 is equal to H
12 - 3 is the height of the triangle in the opened umbrella
12.816 squared - 9 squared = the radius of the open umbrella squared
Radius of open umbrella = square root 83.25 = 9.12 units
Need to highlight the closed umbrella diameter is 9

Average Speed

The plane went the same distance from point A to point B and from point B to point A.

The problem has similar times 100 miles per 1 hour and 80 miles per 1 hour. Let us assume the distance was 800 miles.

On the trip to airport A, it took the airplane driver 8 hours. 800 miles / 8 hours.

On the trip to airport B, it took the airplane driver 10 hours; 800 miles / 10 hours.

Thus the airplane driver flew 1600 total ground miles in 18 hours. This is an average ground speed of 88.89 miles per hour.

HINT (reduces points possible for correct answer to 1): The distance to airport A to B is the same distance as from B to A. How can you convert your miles per hour for the two different trips to include this information?

A factorial is defined as a!=(a)(a-1)(a-2)(a-3)...1 for any positive integer.

For example, 4!=4x3x2x1=24

Find n in the following equation: (6!)(7!)=n!

6 x 5 x 4 x 3 x 2 x 1
7 x 6 x 5 x 4 x 3 x 2 x 1

In red, we have created the numbers that compose 7!

7 x 6 x 5 x 4 x 3 x 2 x 1

Taking 4 and 2 from 7!, we can make 8.

4 x 2
8 x 7 x 6 x 5 x 4 x 3 x 2 x 1

We can use the 3 from 3x2 and the 3 in 7! to make a 9.

6 3
3 x 2

3 x 3 x 8 x 7 x 6 x 5 x 4 x 3 x 2 x 1
9 x 8 x 7 x 6 x 5 x 4 x 3 x 2 x 1

This leaves us from 7! a 2 and a 5. This creates a 10.

5 x 2 x 3 x 3 x 8 x 7 x 6 x 5 x 4 x 3 x 2 x 1

10 x 9 x 8 x 7 x 6 x 5 x 4 x 3 x 2 x 1

10!

Probability Challenge Solution

1. 50%
2. 1/4

Answers from Pilot Peppers

Flight Level 1 Answers:
Why is a nautical mile longer? A nautical mile is based on the circumference of the Earth, specifically defined as one minute (1/60th) of a degree of latitude

Ratio to convert statute miles to nautical miles: approximately 1.15 SM: 1 NM
Ratio to convert nautical miles to statute miles: approximately 0.87 NM: 1 SM
54 SM is approximately 46.92 NM

109 NM is approximately 125.43 SM
Flight Level 2 Answers:

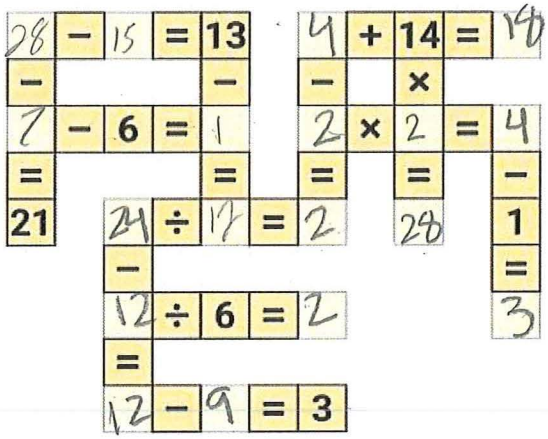
Question: Why did it take longer to get to Moontown than it did to return?
Answer: She was flying with a headwind or into the wind, so it slowed her down.
Question: Based on your answer to the first question, provide calculations and a convincing argument to explain why it took longer to get to Moontown than it did to return.
Answer: The distance to and from are the same. So, rt=rt. Going, you have a headwind, so the 95 kts will be reduced by the unknown headwind. Returning, you have a tailwind, so the 95 kts will be increased by the tailwind. (95 - w)(9/10) = (95 + w)(2/3). Solving using standard algebra, the rate of the wind is approximately 14 knots. Students must express the answer in terms of knots to get credit.

Flight Level 3 Answers:
The travel times are equal; so, D/r = D/r. Therefore, 77/(r - 10) = 93/(r + 10). Standard algebra provides a solution of approximately 106 knots average cruising speed.

TIE BREAKER!

11:30-11:45

When finished, return to envelope and have **pair of students** submit to CRMC staff/volunteers at Printing Press in Breezeway.



1	2	2	4	28	24
2	12	2	7	12	9
18	12	28	15	4	3

Complete the ***Cross Math*** grid by placing the green number tiles in the blank spaces to create a correct number fact. Each green tile will be used once, and all the tiles will be used to complete the grid.