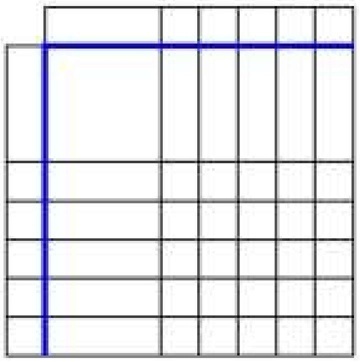
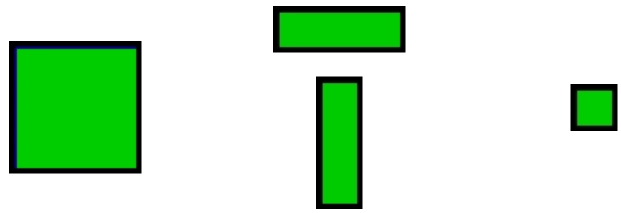
Completing the Square Using Algebra Tiles

1. Use your calculator to graph the equation: $y = x^2 + 6x + 9$. Write the equation in vertex form.

2. Sketch algebra tiles to model the equation $y = x^2 + 6x + 9$.



Recall the values of each of the algebra tiles. The value of the tile is its area. We will only be working with positive (red tiles) representations of algebraic expressions.



x^2 Tile
Area = x • x = x^2 units

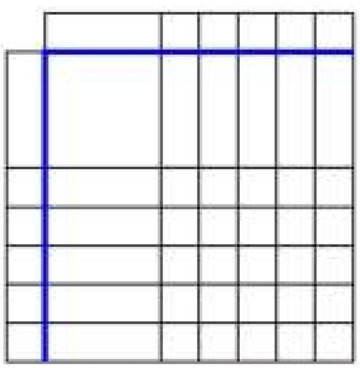
x Tile
Area = 1 • x = x units

Unit Tile
Area = 1 • 1 = 1 unit

Our original equation was written in standard form, $y = ax^2 + bx + c$. Since it is usually much easier to graph a parabola if the equation is in vertex form, $y = a(x-h)^2 + k$, often we try to rewrite the equation from quadratic form into vertex form. We will use algebra tiles to help us understand why this procedure works.

When we are trying to write an equation in vertex form, we need to have a perfect square to make the $(x-h)^2$ part of the equation. When the quadratic equation we are given is not a perfect square, we arrange the parts to form a perfect square, adding what we need or keeping whatever extra pieces we may get. This activity will help you discover how to start the process of forming the perfect square from what you are given.

3. Create a partial square with algebra tiles to represent $x^2 + 2x +$ ______ .



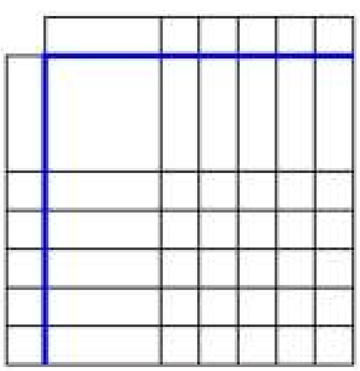
a) How many unit tiles do you need to complete the square?

b) What are the dimensions of the completed square?
L = W =

c) Replace c and ? with numbers to make the statement true:

$$x^2 + 2x + c = (x + ?)^2$$

4. Create a partial square with algebra tiles to represent $x^2 + 4x +$ ______ .



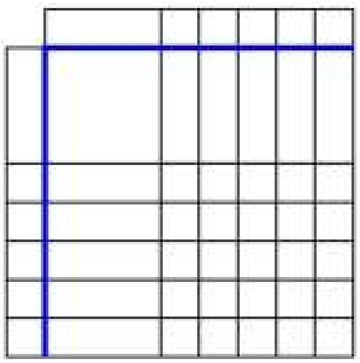
a) How many unit tiles do you need to complete the square?

b) What are the dimensions of the completed square?
L = W =

c) Replace the c and ? with numbers to make the statement true:

$$x^2 + 4x + c = (x + ?)^2$$

5. Create a partial square with algebra tiles to represent $x^2 - 6x +$ ______ .



a) How many unit tiles do you need to complete the square?

b) What are the dimensions of the completed square?
L = W =

c) Fill in the blanks with numbers to make the statement true:
$x^2 - 6x + c = (x + ?)^2$

6. What is the relationship between the coefficient of x and the number of x's you have down one side of your algebra tile diagram?

7. What is the relationship between the number of x's down one side of the algebra tile diagram and the question mark in your perfect square binomial?

8. What is the relationship between the coefficient of x and the question mark in your perfect square binomial?

9. What is the relationship between the question mark of your perfect square binomial and the number of blocks (or units) you had to add to make your diagram a perfect square?

10. In the expression $y = x^2 + bx + c$, how do you use b to get the value of c to form a perfect square? Use the examples above (3-5) to explain your answer.

11. Try these problems—Fill in the missing "c" and then rewrite the trinomial as a perfect square binomial.

a) $x^2 - 10x + c$

b) $x^2 - 4x + c$

c) $x^2 + 12x + c$

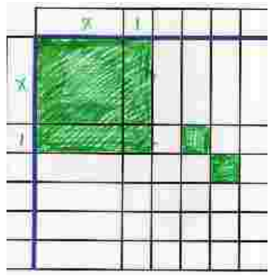
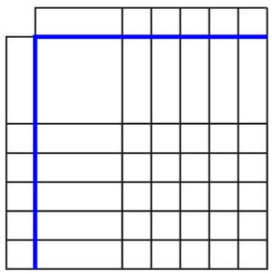
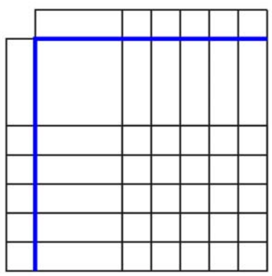
d) $x^2 - 12x + c$

e) $x^2 + 7x + c$

f) $x^2 + bx + c$

Represent each expression by sketching algebra tiles. Try to create a square of tiles. When doing so keep the following rules in mind:

- You may only use **one** x^2-tile in each square.
- You must use **all** the x^2 and x-tiles. Unit tiles are the only ones that can be leftover or borrowed.
- If you need more unit tiles to create a square you have to "borrow" them. The number you borrow will be a negative quantity.

Standard Form	Number of x^2 Tiles	Number of x Tiles	Number of Unit Tiles	Sketch of the Square	Length of the Square	Area of the Square $(\text{Length})^2$	Unit Tiles Left Over (+) Borrowed (-)	Expression Combining Previous Two Columns
$x^2 + 2x + 3$	1	2	3		x+1	$(x+1)^2$	2	$(x+1)^2 + 2$
$x^2 + 4x + 1$								
$x^2 + 6x + 10$								

12. What is the name of the form for the combined expression in the last column?

Convert the following equations from standard form to vertex form by completing the square.

13. $y = x^2 - 8x + 11$

14. $y = x^2 + 16x + 14$

Now let's solve a quadratic equation using Completing the Square.

Steps:	Example: $x^2+6x-14=0$	You try: $x^2+8x-20=0$
a) Move constant term of quadratic to the other side. Write the equation in $ax^2+bx+\_\_=-c+\_\_$ form.	$x^2+6x-14+14=0+14$ $x^2+6x+\_\_=14+\_\_$	
b) Complete the square by adding a constant to both sides.	$x^2+6x+9=14+9$	
c) Rewrite the left side of the equation as a binomial squared and simplify the right side.	$(x+3)^2=25$	
d) Square root both sides.	$x+3=\pm5$	
e) Solve for x.	$x+3=-5$ $x+3=5$ $x+3-3=-5-3$ $x+3-3=5-3$ $x=-8$ $x=2$	

Solve the following equations by completing the square using the steps above.

15. $x^2-4x-32=0$

a)

b)

c)

d)

e)

16. $x^2+8x+7=0$

a)

b)

c)

d)

e)

If the leading coefficient $\neq 1$, we must first divide each term by "a" so that the coefficient of the x^2 term is 1. Then complete the same steps above.

17. $2x^2+4x-3=0$

18. $3x^2+18x+12=0$