**Modeling the Periodic Motion in a Mass-Spring System**

Procedures:

1. Connect the CBR2 to the TI-84 graphing calculator. This should make the calculator start up and launch the CBR2 device interface.
2. Set the sensitivity switch in the Normal position.
3. Go to the Setup menu and enter the following settings:

Sample interval: 0.05 seconds between samples

Number of samples: 60

Experiment duration: 3 seconds

1. Make sure the mass is suspended directly above the motion detector.
2. Hold the base of the stand down to keep it firmly in place.
3. Carefully pull the mass down and release it to start an oscillation. Throughout the oscillation it should remain at least 15 cm (about 6 inches) away from the motion detector.
4. As soon as you are confident that the motion is smooth and only vertical in direction press the key for the START tab on the graphing calculator.
5. The motion of the spring-mass will be sent to your calculator as a graph of position vs. time.
6. If you do not obtain a fairly smooth and consistent graph, press the key for the MAIN tab and repeat steps 6-8. Smaller, smoother oscillations are preferable to larger and erratic oscillations.
7. Once you have a smooth graph press MAIN, QUIT, and OK. The time values will be saved in L1 and the distance measurements in L6.
8. Configure one of the STAT plots and generate a scatter plot of the data you collected.

**Primary Analysis** – use the data collected to determine a function f(x) = A cos B(x-C) + D to model the position of the mass.

1. The model parameter D represents the equilibrium position of the mass (position at rest or average position), as well as the average value. Use [TRACE] to locate the first max and the first min. Record them below. Determine D using these two values. (Explain how).

Position value at first max: \_\_\_\_\_\_\_\_\_\_\_

Position value at first min: \_\_\_\_\_\_\_\_\_\_\_

D =

1. The parameter A represents the amplitude of the oscillations (farthest distance away from the equilibrium position). Use the first max and first min to compute A.
2. The parameter B determines the frequency of the oscillations, and it can be determined from the observed period (cycle length). The observed period of the motion of the spring can be found by using [TRACE] to find the x-values at the first two max's on your graph and then computing their difference. Once you have found the observed period for the graph, B can be filled in for the function formula above by using.

Observed period = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

B = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

1. It remains to determine a value of the parameter C. The un-transformed cosine function achieves its first maximum at x = 0. Since C represents a right shift, we can determine its value from the x-coordinate of the first max.

C = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

1. Record your function formula below, enter the equation in the “Y =” screen of the calculator, and graph it with your scatter plot.

f(x) = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

**Extending the Analysis**

1. ***Damped motion and products of two functions.*** If you run the oscillation for a longer duration (10 seconds), you will see that the oscillations are actually decreasing in amplitude due to natural damping in the system.
   1. Use the coordinates of the peaks and regression to generate an exponential function.
   2. Construct a model for the damped motion as a product of the exponential function and the cosine function obtained in the steps of the Primary Analysis, .
2. ***Bring in considerations about velocity and the first derivative of the position function.***
   1. Compare the derivative of f(x) with the velocity data stored in L7 as described in the following. Use differentiation rules to compute the first derivative of v(x) to obtain the velocity function. Create a scatter plot for the velocity data using L1 as the XLIST and L7 as the YLIST. Graph your equation for the velocity along with the scatter plot. How well do the two graphs match? Why might there be some discrepancies? Compare the graph of your derivative function u(x) with the graph of the position function f(x). When f(x) has a maximum, what can you say about the graph of v(x)? When f(x) has a minimum, what can you say about the graph of u(x)?
   2. As an alternative to part a, you can define the derivative function without knowing/using the differentiation rules – the calculator can compute the derivative numerically using NDERIV. To define the derivative of a function stored as Y1, choose another Y variable, such as Y2. While the cursor sits in the expression for Y2 in your calculator, select the MATH button. Scroll down the menu until you see NDERIV( and press ENTER. This will open up a template like the following:

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To define Y2 as the function you get as the derivative of the function stored in Y1, fill the template in as follows:

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Now the same questions posed in part a can be addressed by graphing Y2.