Title:	**Grade:** 8	**BIG Idea:**
Lines and Transversals: An Introductory Lesson	**Authors**: Hope Phillips	Geometry: Lines Cut by a Transversal

Prior Knowledge Needed:

- how to determine and identify acute, right, obtuse, supplementary, and straight angles
- how to identify non-parallel lines
- how to identify parallel lines
- how to identify perpendicular lines
- how to solve and evaluate two-step algebraic equations

GPS Standards:

M8G1. Students will understand and apply the properties of parallel and perpendicular lines and understand the meaning of congruence.

a. Investigate characteristics of parallel lines both algebraically and geometrically.

b. Apply properties of angle pairs formed by parallel lines cut by a transversal.

M8P2. Students will reason and evaluate mathematical arguments.

Objectives:

1. Given a pair of non-parallel lines cut by a transversal, students will identify the following special angle pairs:

   - Corresponding
   - Alternate interior
   - Alternate exterior
   - Vertical
   - Linear pair

2. Given a pair of parallel lines cut by a transversal, students will identify the following special angles pairs as congruent:

   - Corresponding
   - Alternate interior
   - Alternate exterior
   - Vertical
   - Linear pair

3. Given a pair of parallel lines cut by a transversal and an angle written as a numerical or algebraic expression, students will determine the numerical values of all the angles.

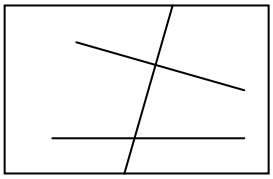
Materials:

Patty paper, clear transparencies, or wax paper (*note: Patty paper can be bought at food supply warehouses.)

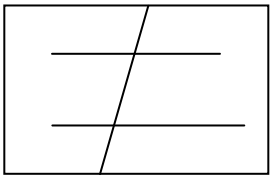
PowerPoint presentation
Investigating Lines Cut by Transversals: Test Your Knowledge student handout
Rulers
Paper/pencil

Task:

What are the special angle pairs given the following sets of lines?



Non-parallel



Parallel

Description and Teacher Directions:

Discuss the names of the angle pairs and where they are located (refer to slide with non-parallel lines cut by a transversal). A transversal is a line interesting two or more lines.

Pass out rulers. Ask students to copy the contents of the slide on their papers. Students do not need an exact copy of the slide as long as their lines do not intersect and they have the same numbered angles identified.

Discuss each set of angle pairs, including supplementary angles. Once each pair has been identified, ask students to consider whether each pair is congruent. Students should use patty paper to copy one of each of the angle pairs. Then, students should rotate the patty paper to lie on top of the second angle of the pair.

For example, angle 1 should be copied on patty paper and rotated to lie on top of angle 5 to determine whether corresponding angles of non-parallel lines are congruent.

For example, Angle 4 should be copied and rotated to lie on top of angle 6 to determine if alternate interior angles of non-parallel lines are congruent.

Protractors are not needed to measure the angles because of the patty paper. However, references to acute and obtuse angles can help students compare the general measures of the angle pairs.

Teacher Commentary:

The comments below are from the teacher who piloted this lesson.

This is a great introductory lesson to be used when just beginning the concepts. Students may bring up additional vocabulary terms during class discussion. These include *acute, obtuse,* and *complementary angles.*

On the student handouts, the questions involving algebraic expressions can be challenging for students. I did not tell them anything. They figured it out after reasoning for themselves. I was really proud of them for that.

The PowerPoint is very well laid out. My students wanted interaction with the SmartBoard (which I did) so they could move around some. They love showing off at the board.

The rubric was easy and quick to use for grading. Wax paper works well if patty paper is not available.

Continue copying angles on patty paper with each of the four types of angle pairs (i.e. corresponding, alternate interior, alternate exterior, and vertical). Name all of the sets of angles that comprise each category of angle pairs (see discussion below). **Linear Pairs:** two adjacent angles that are supplementary; together they form a straight angle whose measure is 180° Angles 1 & 2; 3 & 4; 5 & 6; 7 & 8; 1 & 4; 2 & 3; 5 & 8; 6 & 7 **Vertical angles**; two non-adjacent angles (they share no common side) formed by the intersection of two lines/segments; share a vertex in common Angles 1 & 3; 2 & 4; 5 & 7; 6 & 8 **Corresponding angles**: angles that lie in the same position relative to the transversal and other two lines Angles 1 & 5; 4 & 8; 2 & 6; 3 & 7 For example, angles 1 and 5 both lie to the right of the transversal and above the lines **Alternate interior angles**; angles that lie on opposite sides of the transversal and in between the other two lines Angles 4 & 6; 3 & 5 For example, angle 3 lies to the left of the transversal, while angle 5 lies on the right. Both angles lie between the two lines. **Alternate exterior angles**; angles that lie on opposite sides of the transversal and are outside of the two lines Angles 1 & 7; 2 & 8 For example, angle 1 lies to the right of the transversal and outside of one of the lines.	

Angle 7 lies to the left of the transversal and outside of one of the lines. Students should come to the conclusion that <u>vertical angles are the only special pair of congruent angles when non-parallel lines are cut by a transversal.</u> Ask students to copy the diagram on their papers of **parallel** lines cut by a transversal. The use of notebook paper will help ensure the lines are parallel. The students do not need an exact copy of the diagrams as long as their lines are parallel and have the same numbered angles identified. Using patty paper again, students should determine whether the special angle pairs that lie on parallel lines are congruent. <u>All special angle pairs are congruent.</u> Several geometry theorems exist concerning parallel lines cut by a transversal. These theorems begin with the assumption that the two lines cut by the transversal are, in fact, parallel. They begin like this – "If two parallel lines are cut by a transversal, then…" corresponding angles are congruent. **or** alternate interior angles are congruent. **or** alternate exterior angles are congruent. The converses of each of these theorems are important to consider, also. The focus here is not on the assumption of parallel lines but, rather, the assumption of congruent angle pairs. These theorems begin like this--	

If two lines have been cut by a transversal and…

corresponding angles are congruent

or

alternate interior angles are congruent

or

alternate exterior angles are congruent

then…

the lines are parallel.

Students need to consider both theorems to understand what can be inferred from a situation where parallel lines cut by a transversal exist and/or where congruent angle pairs exist. Understanding the converse becomes the basis for constructing parallel lines.

The **solutions** for the questions at the end of the PowerPoint are shown below.

Slide #15:

$\angle 1$ and $\angle 5$ are corresponding angles. Because the lines are parallel, the measures of the angles are equal. So, the measure of $\angle 5$ is also 60°.

$\angle 3$ measures 60° because $\angle 1$ and $\angle 3$ are vertical angles.

$\angle 7$ measures 60° because $\angle 1$ and $\angle 7$ are alternate exterior angles.

$\angle 2$ measures 120° because $\angle 2$ and $\angle 1$ form a linear pair.

$\angle 2$ and $\angle 4$ measures 120° because they form vertical angles.

$\angle 6$ measures 120° because $\angle 6$ and $\angle 4$ are alternate interior angles.

$\angle 8$ measures 120° because $\angle 4$ and $\angle 8$ form corresponding angles.

*There are multiple solution paths to arrive at the measure of the angles listed above. This explanation is only one way to approach the problem.

Slide #16:

m ∠6 = 140°
m ∠1 = 40°
m ∠2 = 140°
m ∠3 = 40°
m ∠4 = 140°
m ∠5 = 40°
m ∠6 = 140°
m ∠7 = 40°
m ∠8 = 140°

Slide #17:

(5x + 60) and (3x - 40) are supplementary, meaning they form a straight angle/linear pair. Together their sum measures 180°.

$5x + 60 + 3x - 40 = 180$
$8x + 20 = 180$
$-20 = -20$
$\frac{8x}{8} = \frac{160}{8}$
$x = 20$

To find the measure of each angle, substitute the value of "x" back in the algebraic expressions.

$5(20) + 60 = 160°$ $3(20) - 40 = 20°$

$160 + 20 = 180°$

Slide #18:

(2x – 15) and (x + 12) form alternate exterior angles. Because the lines are parallel, the measures of these angles are equal. To solve, set them equal to each other.

$2x - 15 = x + 12$
$-x = -x$
$x - 15 = 12$
$+15 = +15$
$x = 27$

Substituting the value of “x” in the expression (2x – 15), the measure of the angle is 39°. Substituting the value of “x” in the expression (x + 12), the measure of the angle is 39°, also. The measurement is the same because the angles form alternate exterior angles.

m∠2 = 141°
m ∠3 = 39°
m ∠4 = 141°
m ∠5 = 39°
m ∠6 = 141°
m ∠8 = 141°

Slide #19:

(3x – 120) and (2x - 60) form alternate interior angles. Because the lines are parallel, the measures of these angles are equal. To solve, set them equal to each other.

$$\begin{aligned} 3x - 120 &= 2x - 60 \\ -2x \quad &= -2x \\ \hline x - 120 &= -60 \\ +120 \quad &= +120 \\ \hline x &= 60° \end{aligned}$$

Substituting the value of “x” in the expression (3x -120), the measure of the angle is 60°. Substituting the value of “x” in the expression (2x - 60), the measure of the angle is 60°, also. The measurement is the same because the angles form alternate exterior angles.

m∠1= 60°
m∠2 = 120°
m∠4 = 120°
m∠6 = 120°
m∠7= 60°
m∠8 = 120°

Modifications/Extensions:

Modifications: Copy the handout and PowerPoint slides for students who cannot draw the diagrams.

Resources:

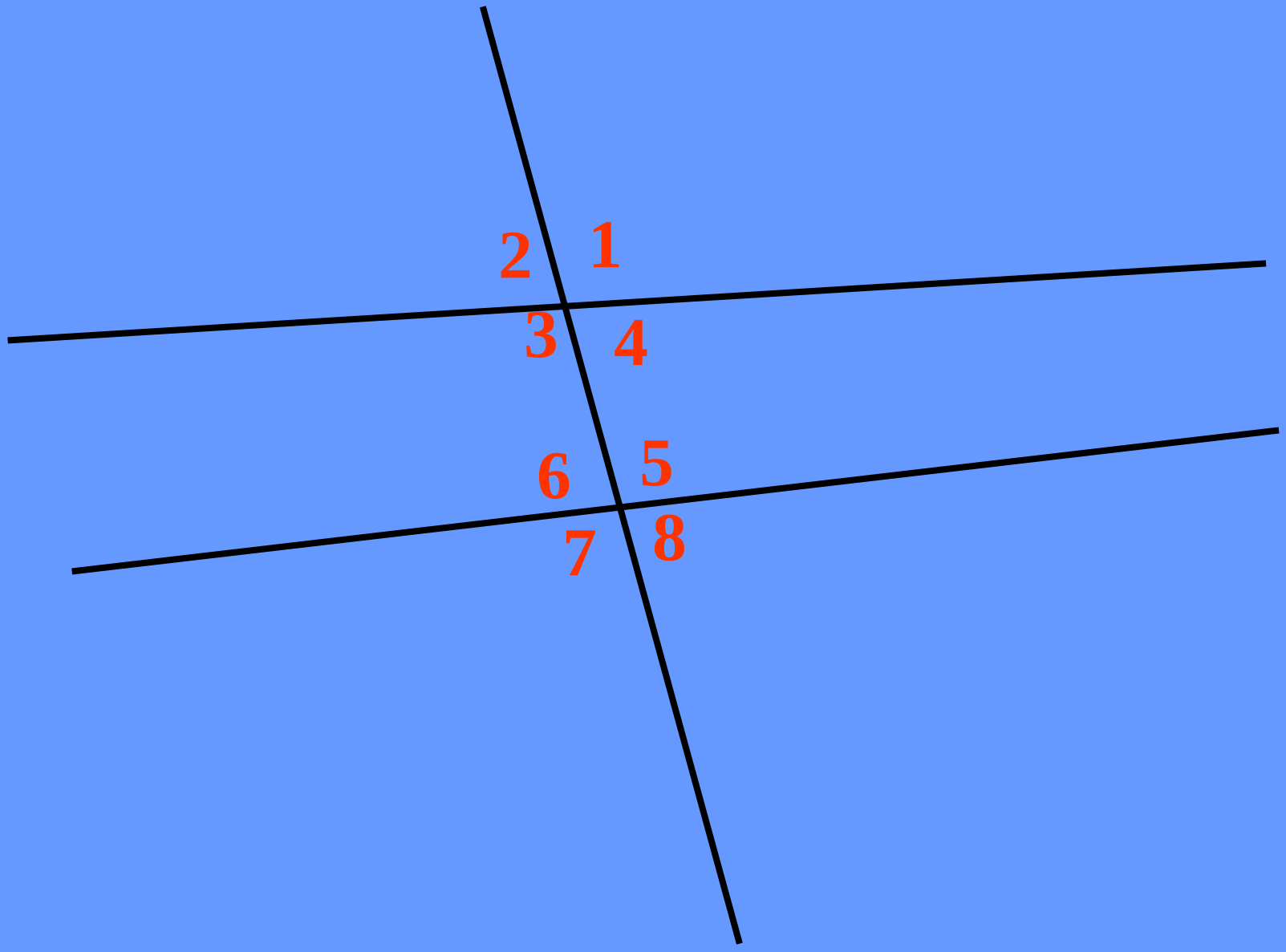
PowerPoint presentation

LINES CUT BY A TRANSVERSAL

Vocabulary

- PARALLEL
- TRANSVERSAL
- ANGLE
- CONGRUENT
- VERTICAL ANGLES
- CORRESPONDING ANGLES
- ALTERNATE INTERIOR ANGLES
- ALTERNATE EXTERIOR ANGLES
- LINEAR PAIR
- STRAIGHT ANGLES

Non-Parallel Lines Cut by a Transversal



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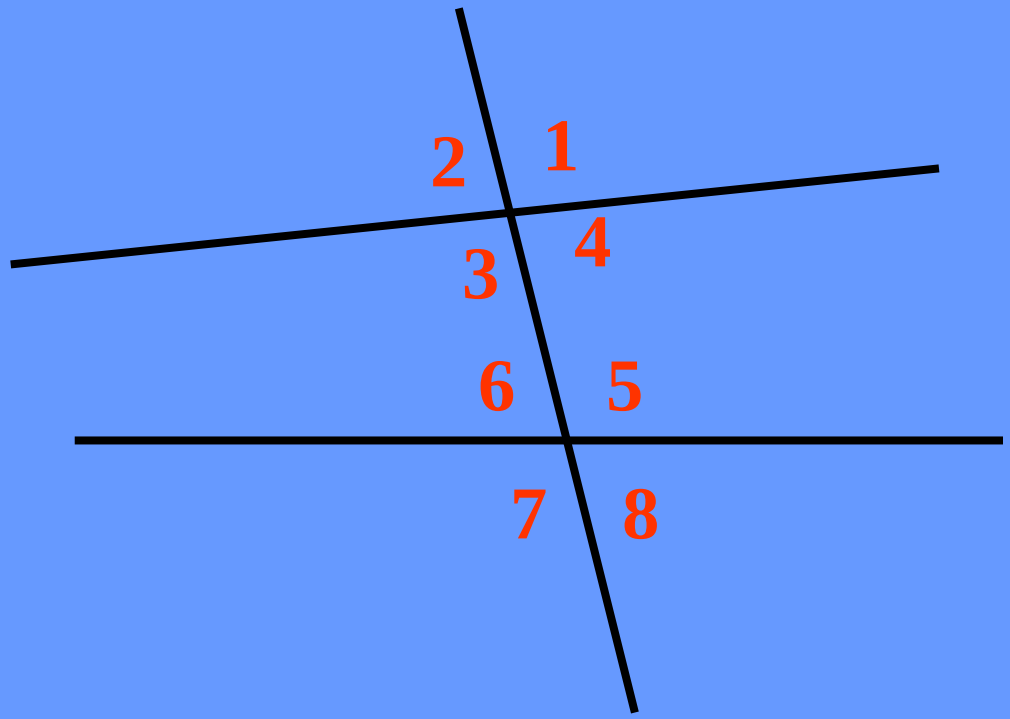
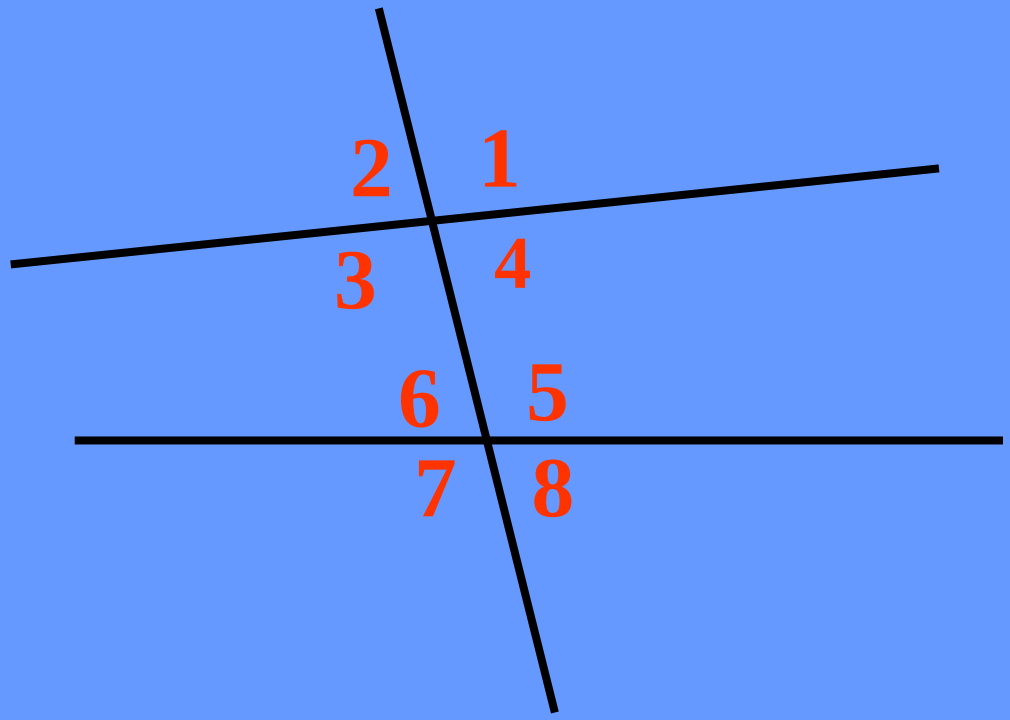
Non-parallel lines cut by a transversal



∠1 and ∠2 are a LINEAR PAIR

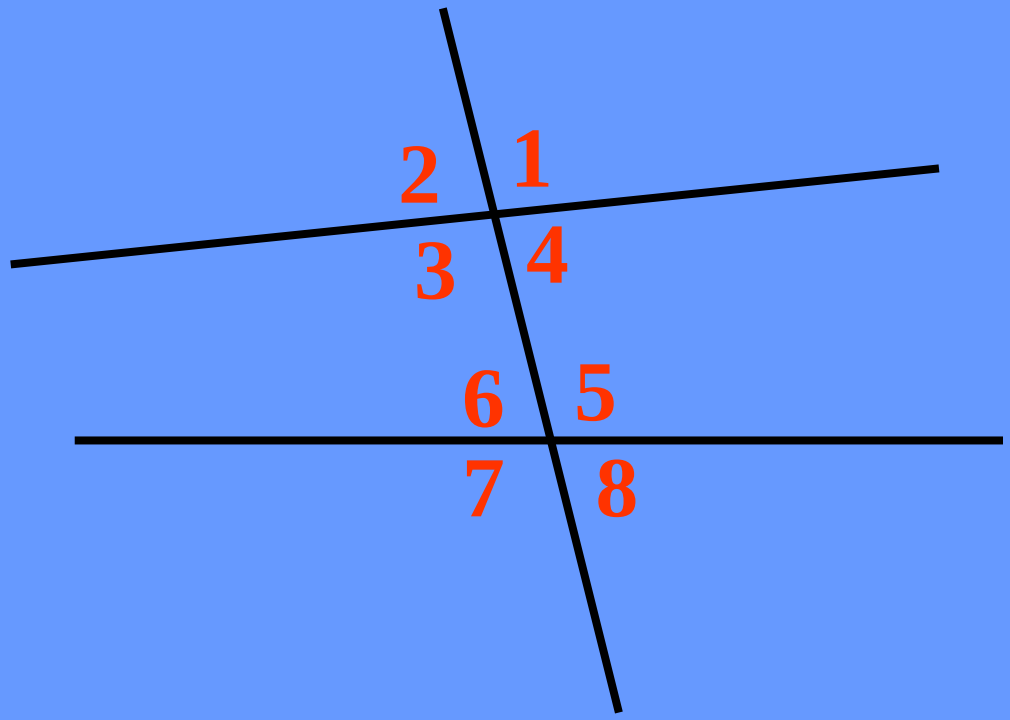
They form a straight angle measuring 180 degrees.
Since their sum is 180°, they are supplementary.

Non-parallel lines cut by a transversal



Name other linear pairs.

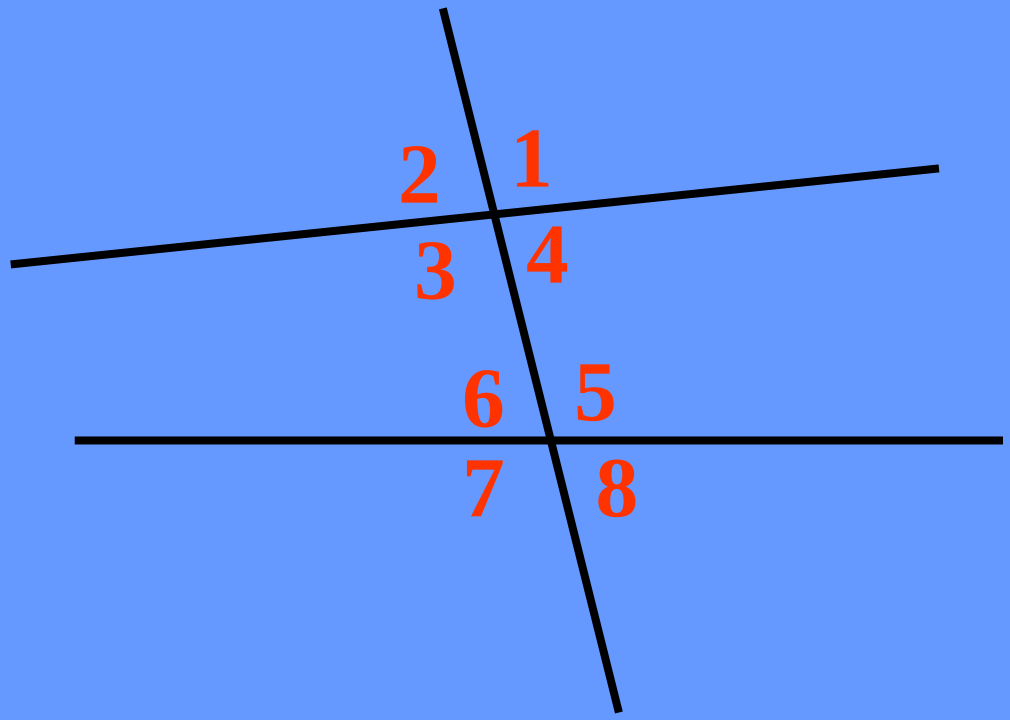
Non-parallel lines cut by a transversal



∠ 1 and ∠3 are VERTICAL ANGLES

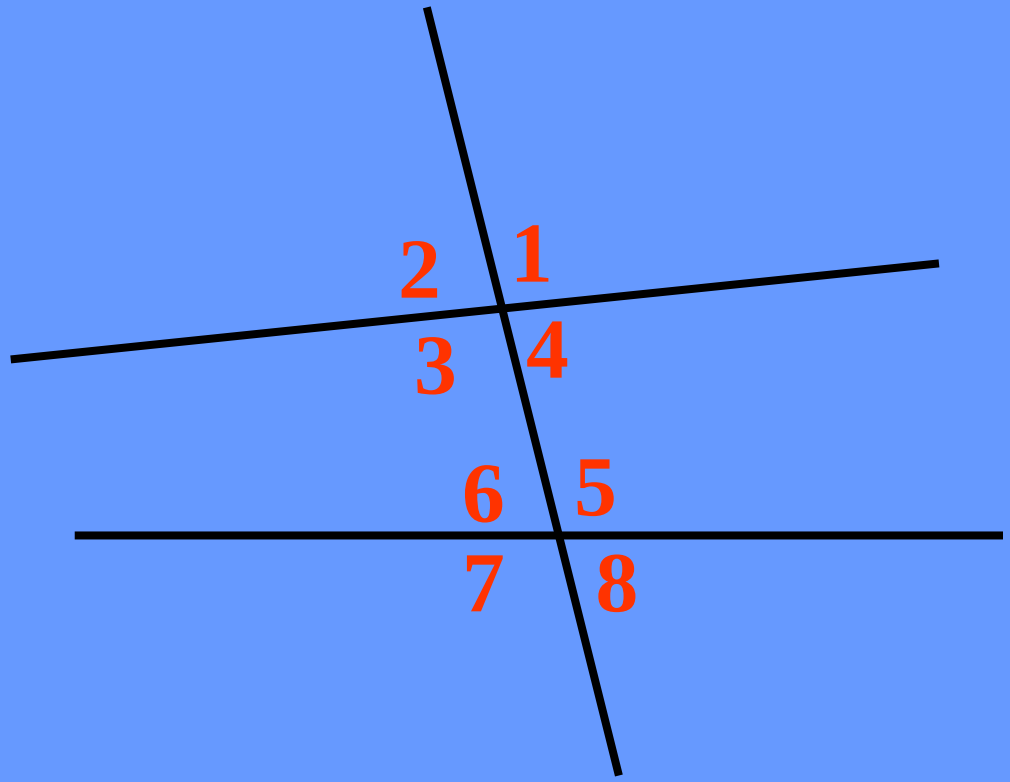
They are congruent, so m <1 = m <3.

Non-parallel lines cut by a transversal



Name other vertical angles.

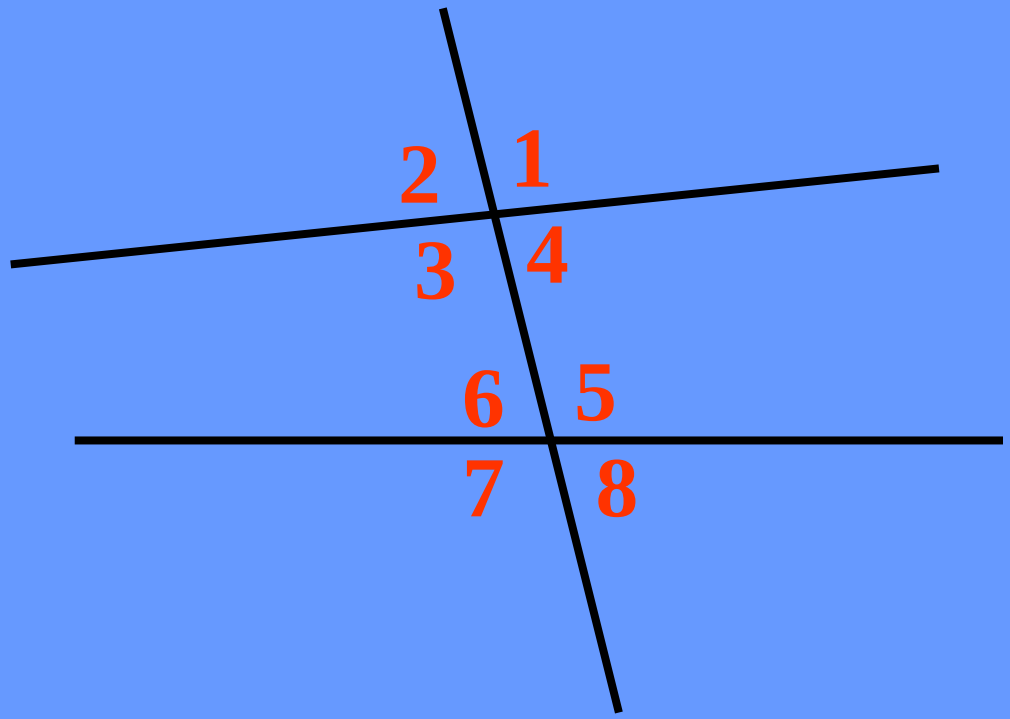
Non-parallel lines cut by a transversal



∠ 1 and ∠ 5 are CORRESPONDING ANGLES

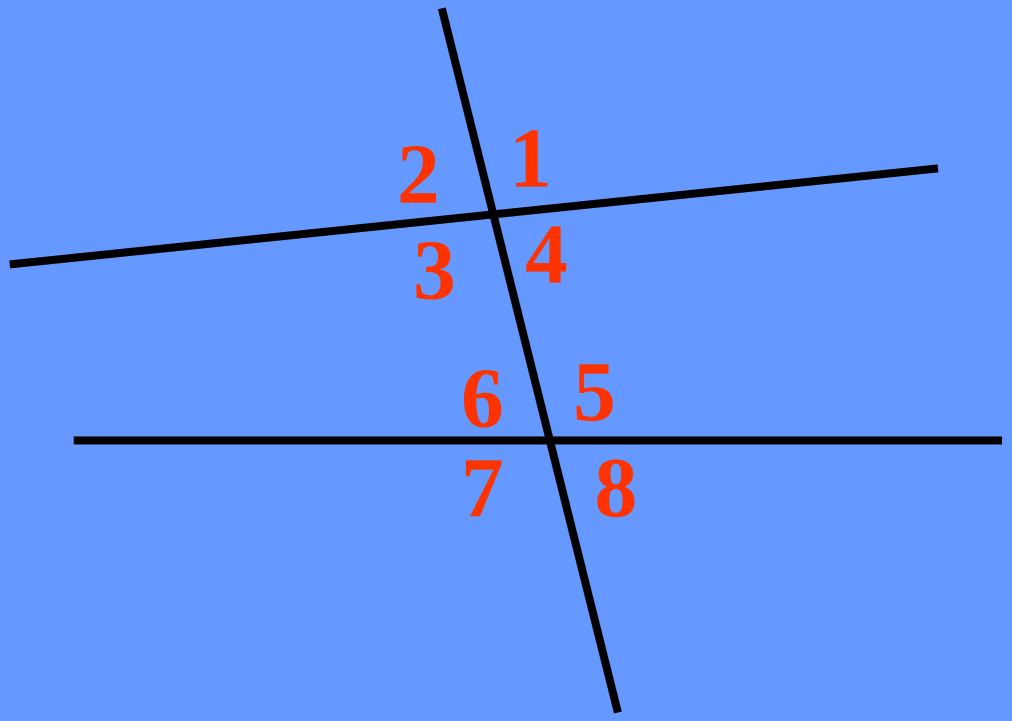
Corresponding angles occupy the same relative position on the same side of the transversal.

Non-parallel lines cut by a transversal



Name other corresponding pairs.

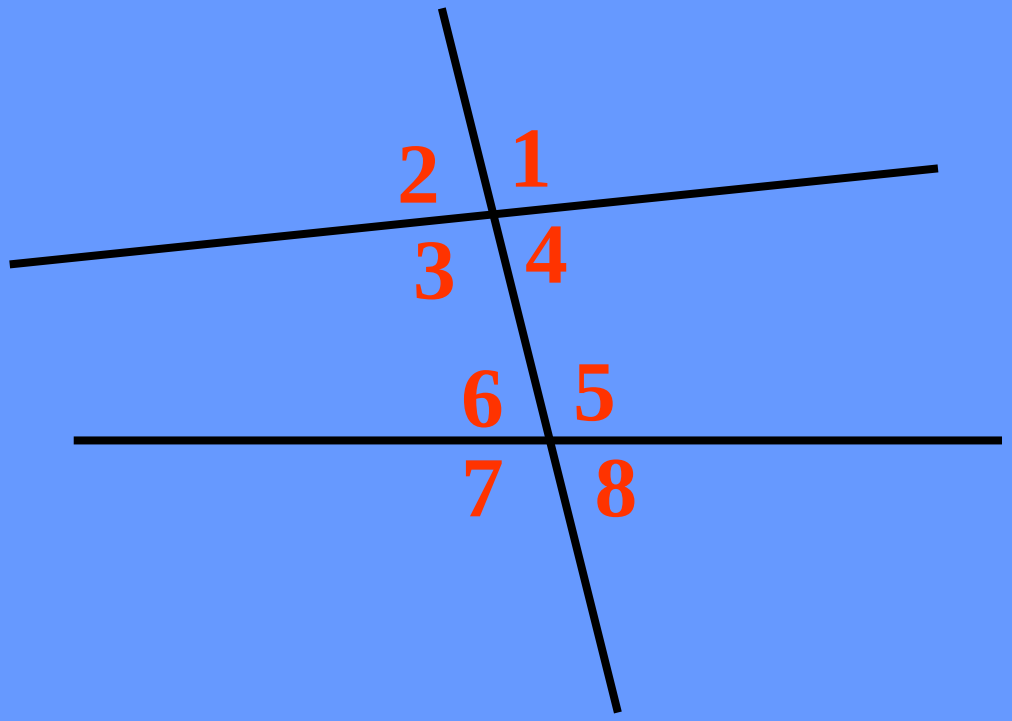
Non-parallel lines cut by a transversal



∠ 4 and ∠ 6 **are ALTERNATE INTERIOR ANGLES**

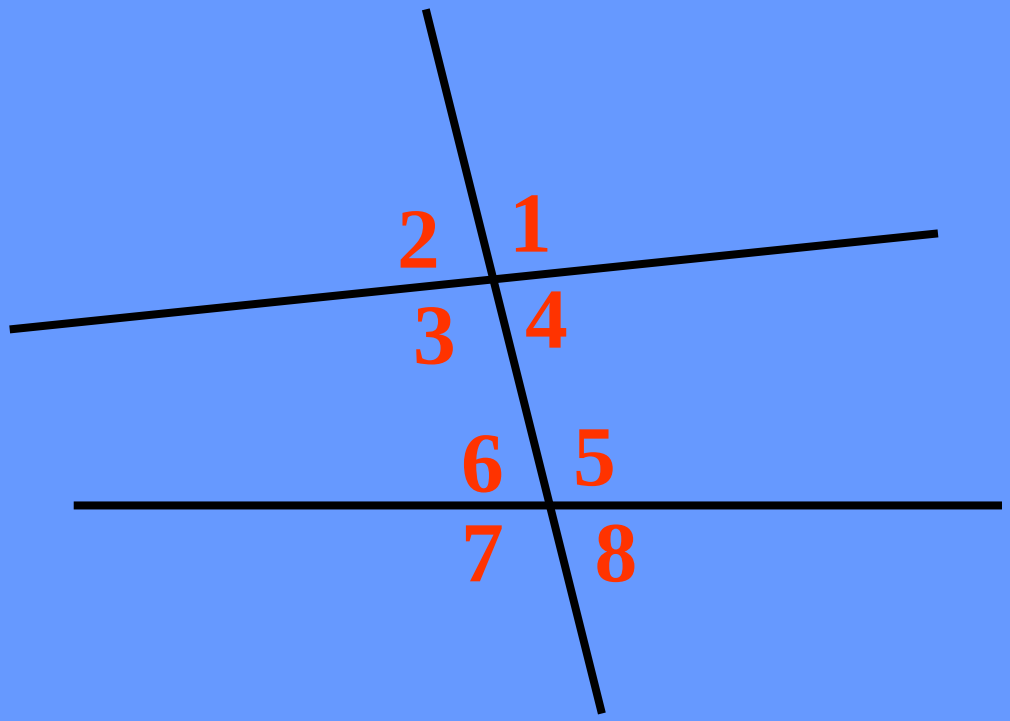
Alternate Interior Angles are on the inside of the two parallel lines and on opposite sides of the transversal.

Non-parallel lines cut by a transversal



Name other alternate interior angles.

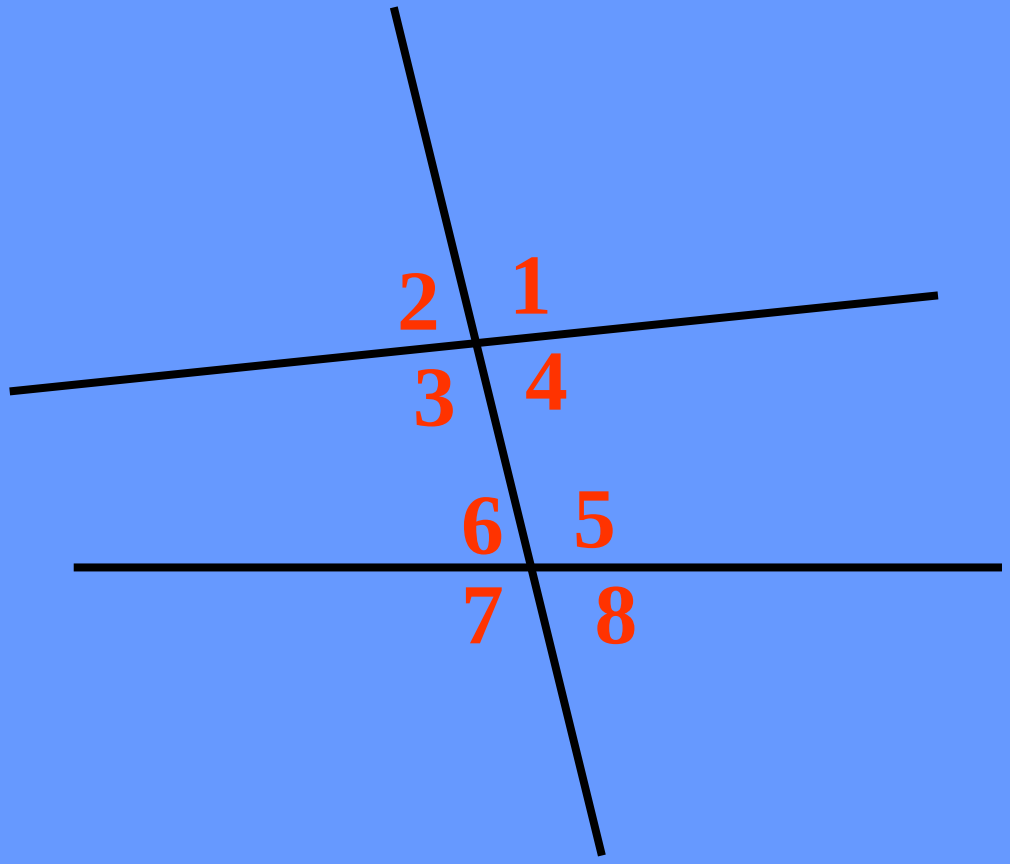
Non-parallel lines cut by a transversal



∠ 2 and ∠ 8 **are** **ALTERNATE EXTERIOR ANGLES**

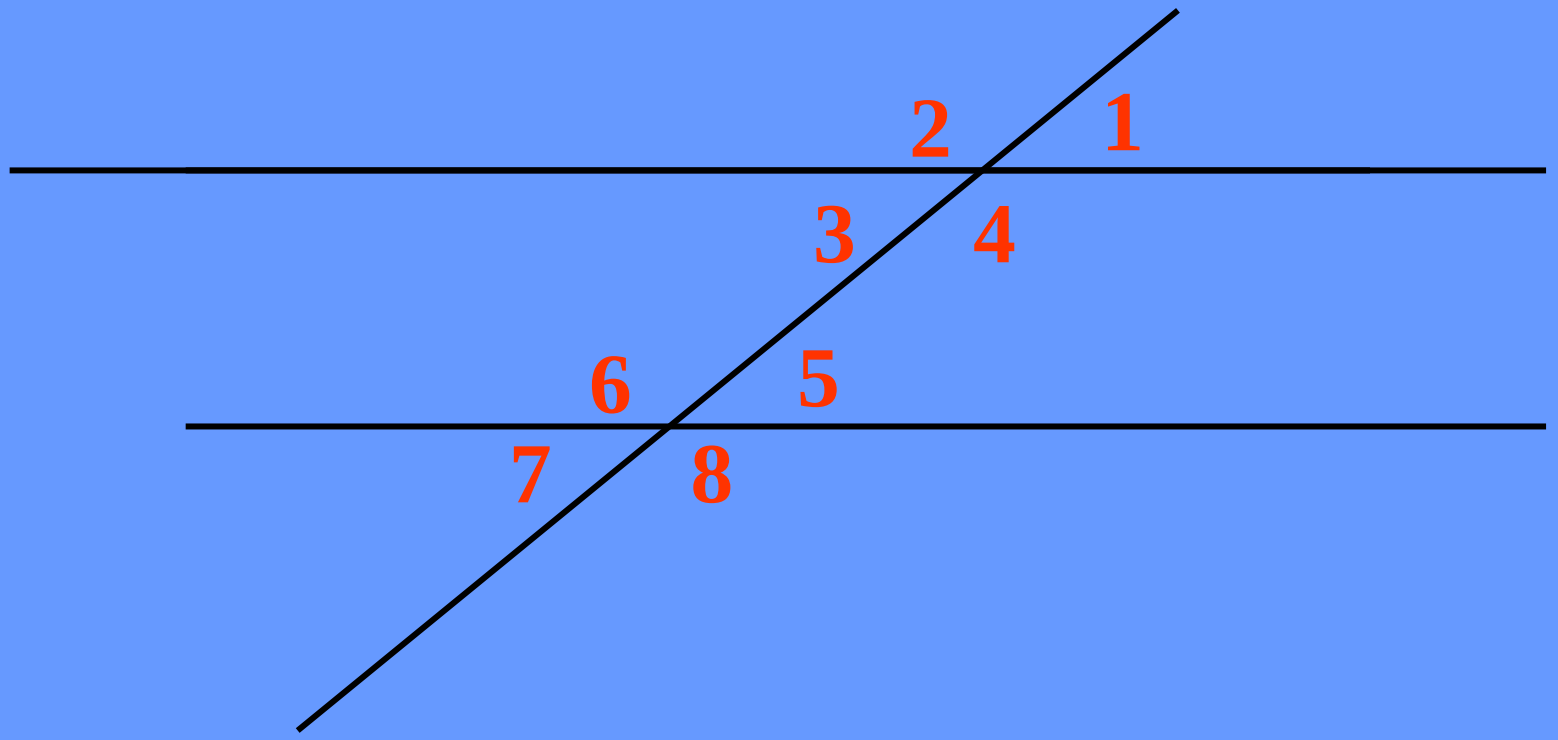
Alternate Exterior Angles are on the outside of the two parallel lines and on opposite sides of the transversal.

Non-parallel lines cut by a transversal



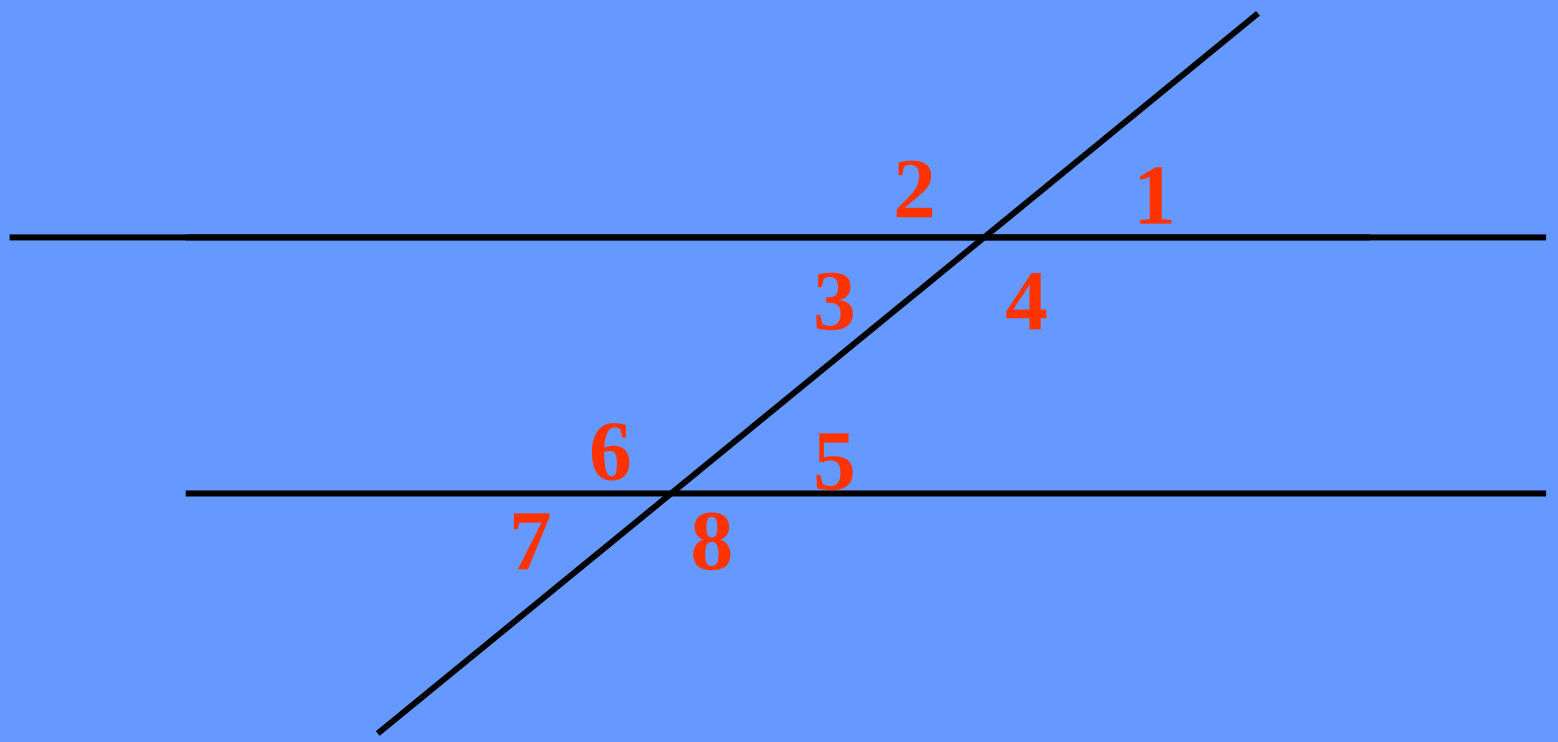
Name other alternate exterior angles.

Parallel lines cut by a transversal



Are any of the special angle pairs congruent?

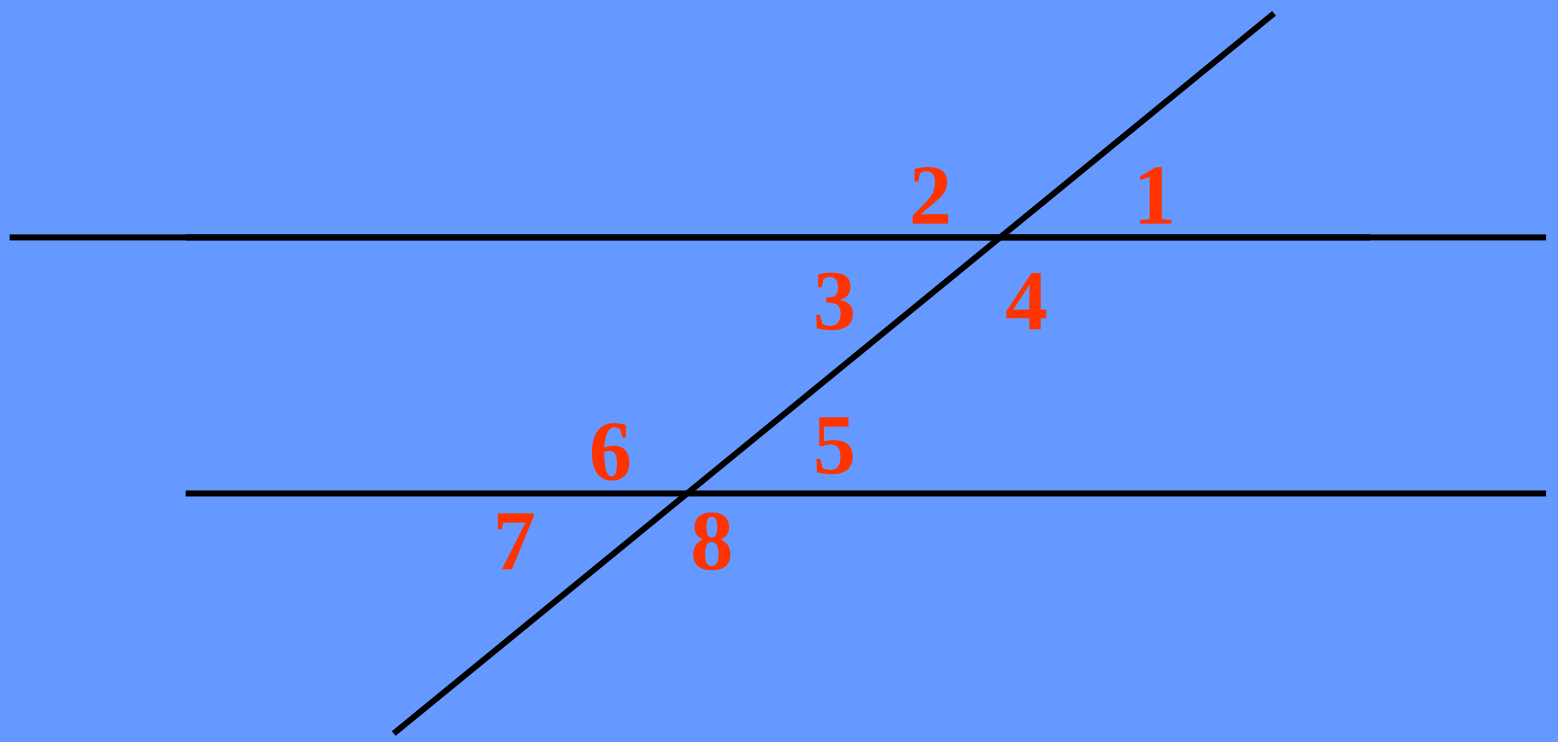
Parallel Lines Cut by a Transversal



The m $\angle$ 1 is 60°.

What are the measures of the other numbered angles?

Parallel Lines Cut by a Transversal

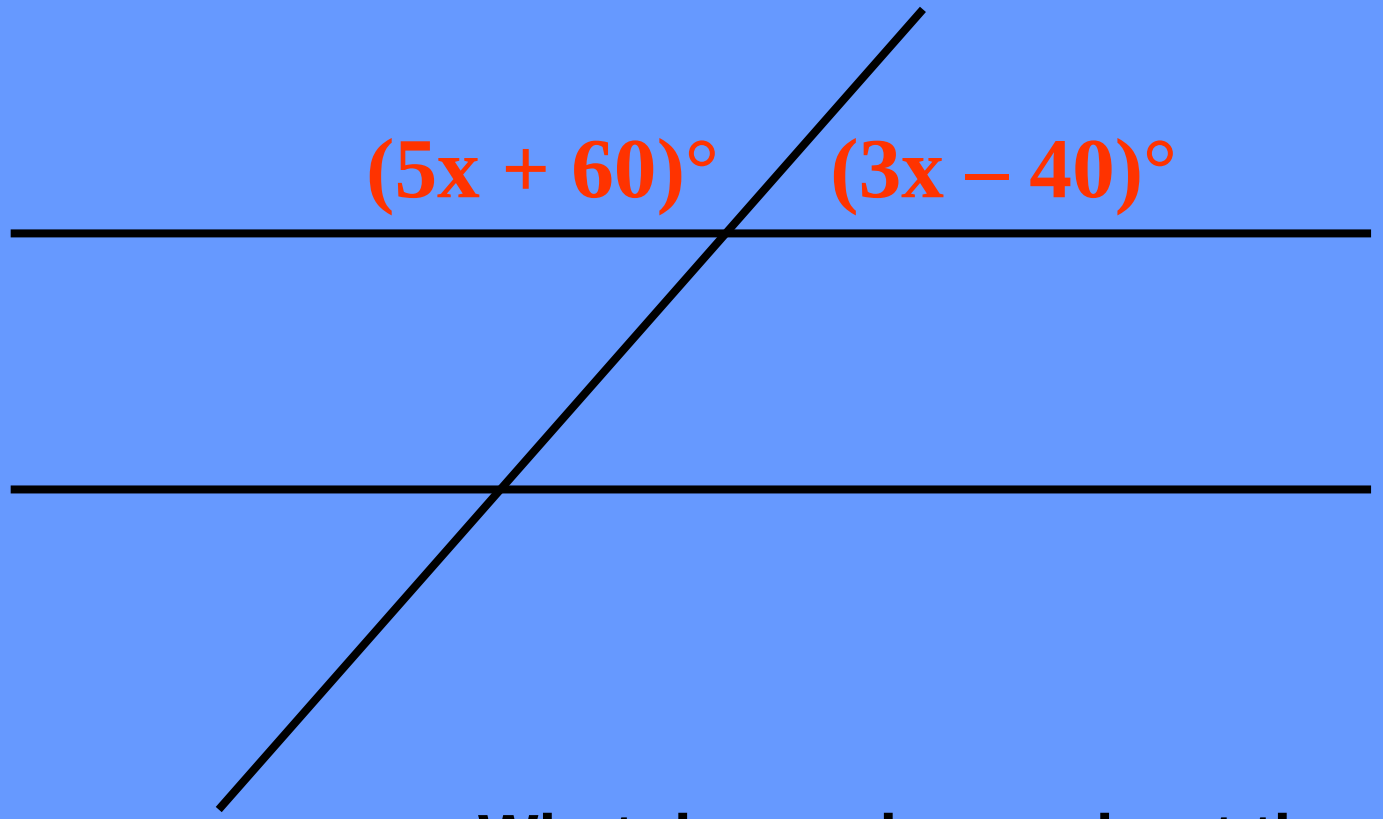


The m ∠ 6 is ∠ 140°.

What are the measures of the other numbered angles?

Parallel Lines Cut by a Transversal

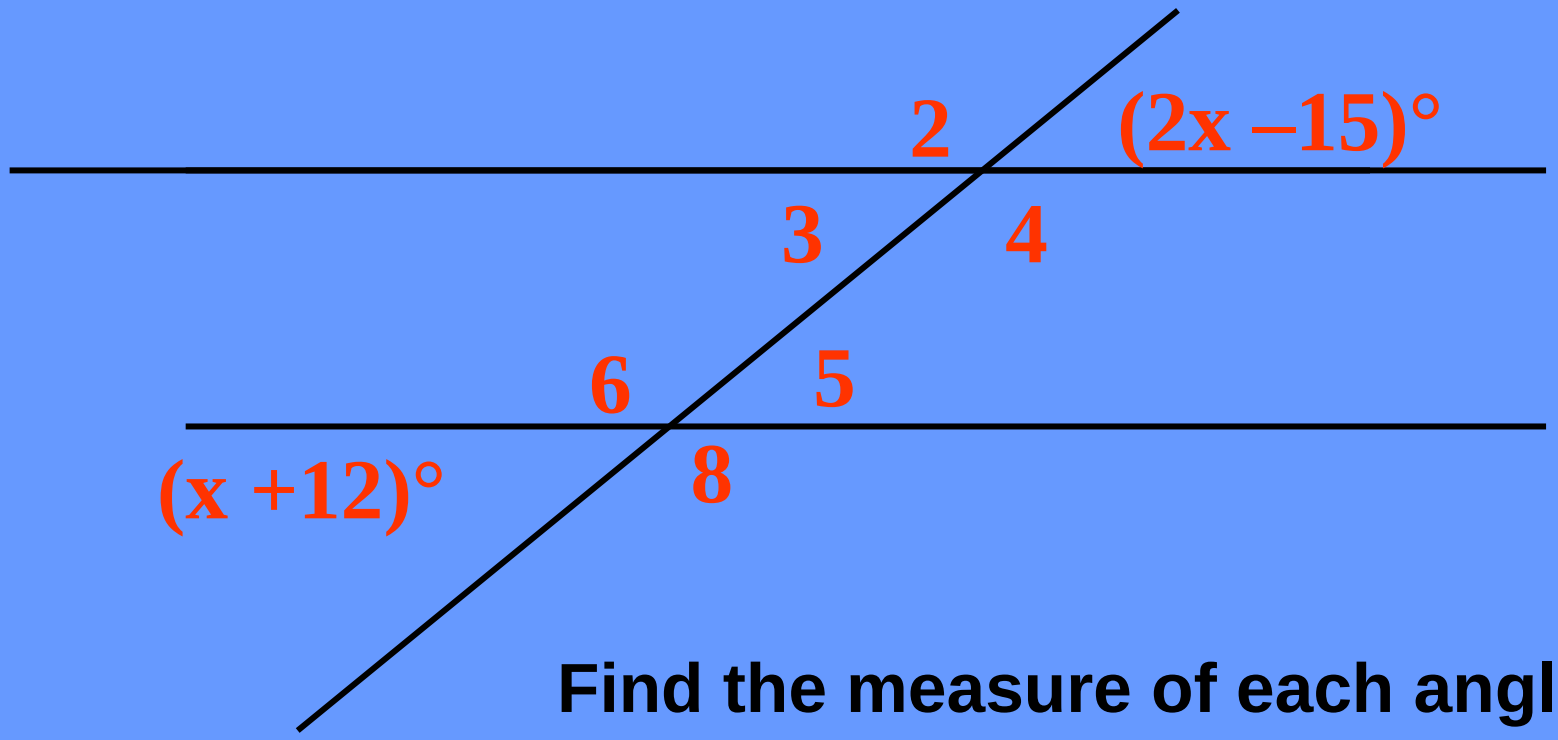
What are the measures of the angles below?



What do you know about the angles?
Write the equation.
Solve for x.
Find the measure of each angle.

Parallel Lines Cut by a Transversal

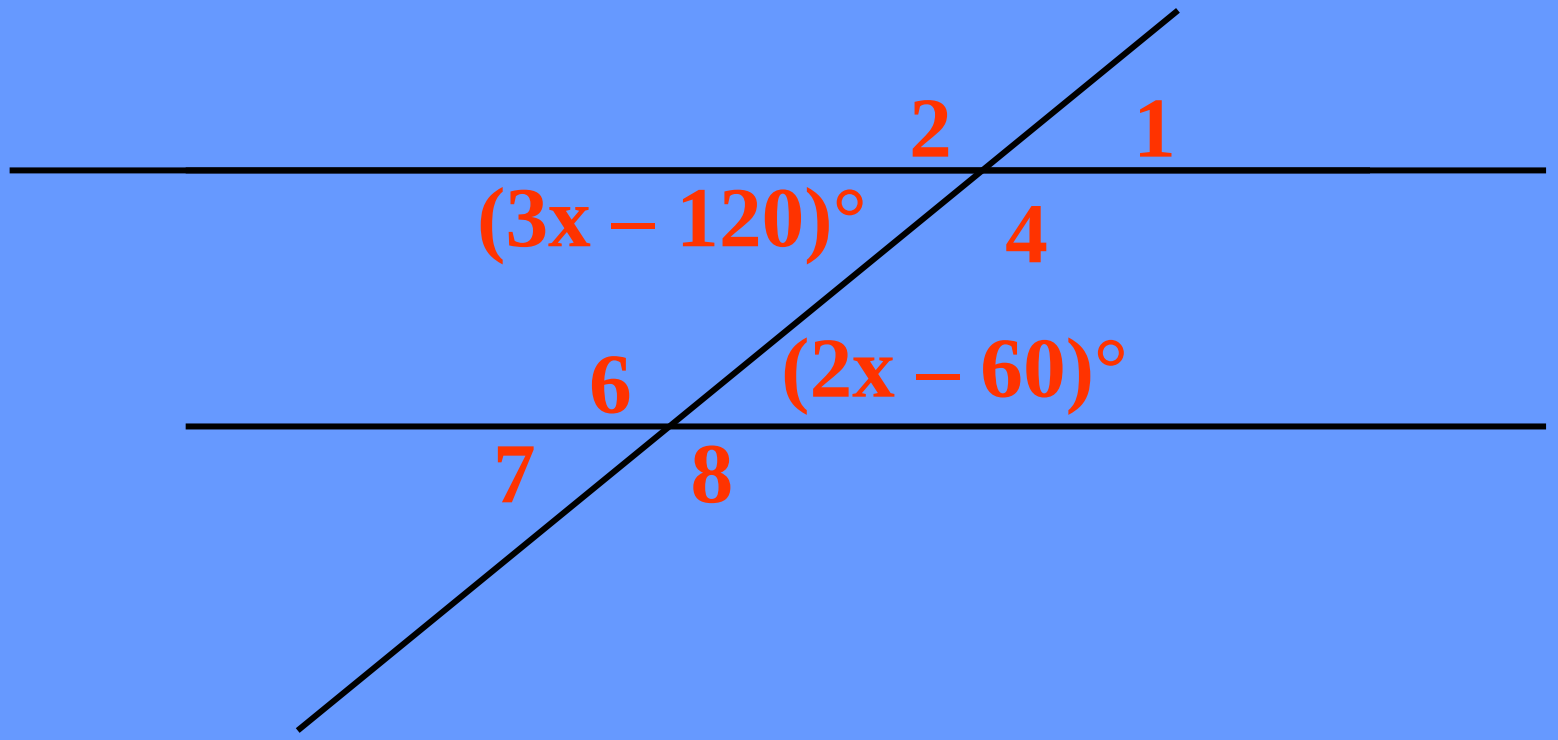
What are the measures of the angles below?



Find the measure of each angle.

Parallel Lines Cut by a Transversal

What are the measures of the angles below?



Find the measure of each angle.

Lines Cut by a Transversal Task – Teacher Checklist

1. Student can correctly identify special angle pairs.
2. Student correctly finds the measure of all angles given certain angles written as numerical expressions.
3. Student correctly applies the appropriate relationship to determine the value of the supplementary angles.
4. Student correctly finds the measure of all angles given certain angles written as algebraic expressions.
5. Student can solve two-step algebraic equations.
6. Student can correctly articulate their reasoning in writing.

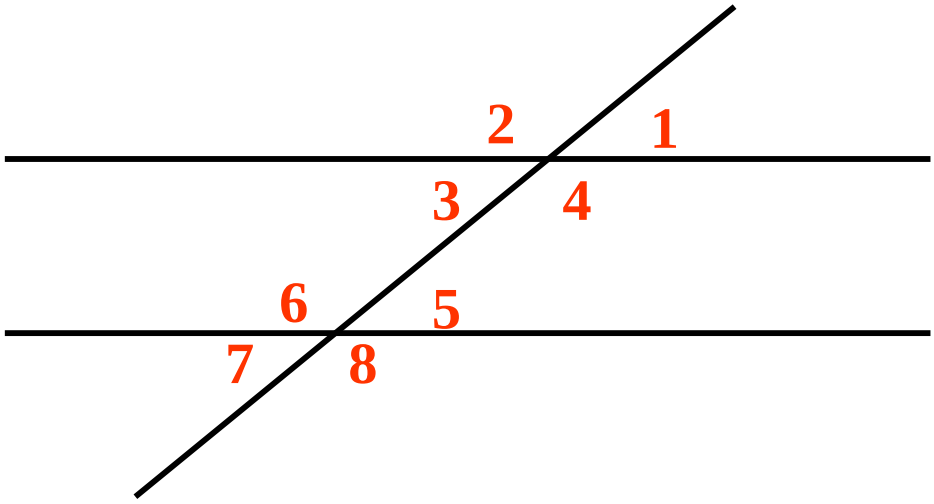
	Question						
NAME	**1**	**2**	**3**	**4**	**5**	**6**	**Comments**
	(+) = > 75% of the time **(-)** = < 75% of the time **(blank space)** = no attempt						

Investigating Lines Cut by Transversals
Test Your Knowledge

Directions: Fill in the blanks below.

1.

Parallel Lines Cut by a Transversal



Name the angle that corresponds to ∠1 _____.

Name the angle that is alternate interior to ∠3 _____.

Name the angle that is alternate exterior to ∠8 _____.

Name the angle that corresponds to ∠8 _____.

Name the angle that is alternate exterior to ∠7 _____.

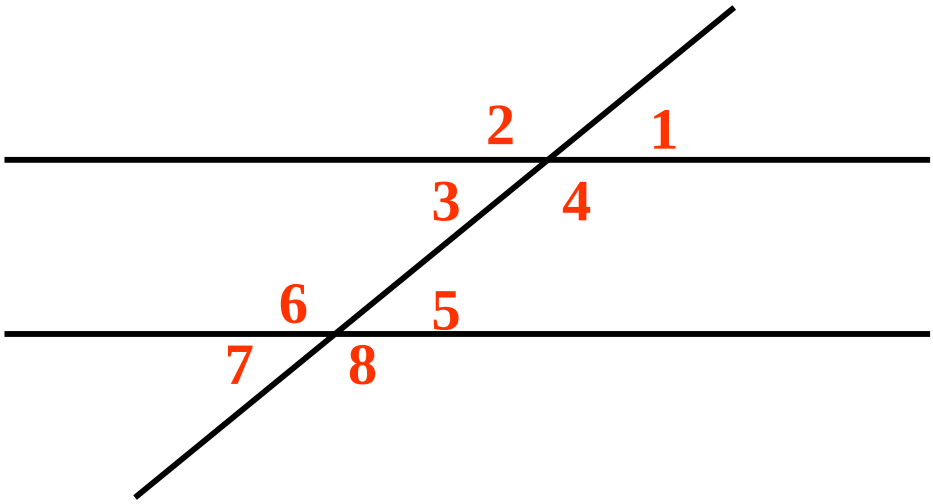
Name the angle that is alternate interior to ∠6 _____.

Investigating Lines Cut by Transversals
Test Your Knowledge

Directions: Show your work and explain your reasoning using complete sentences.

2.

Parallel Lines Cut by a Transversal



The m $\angle$ 1 is 60°.

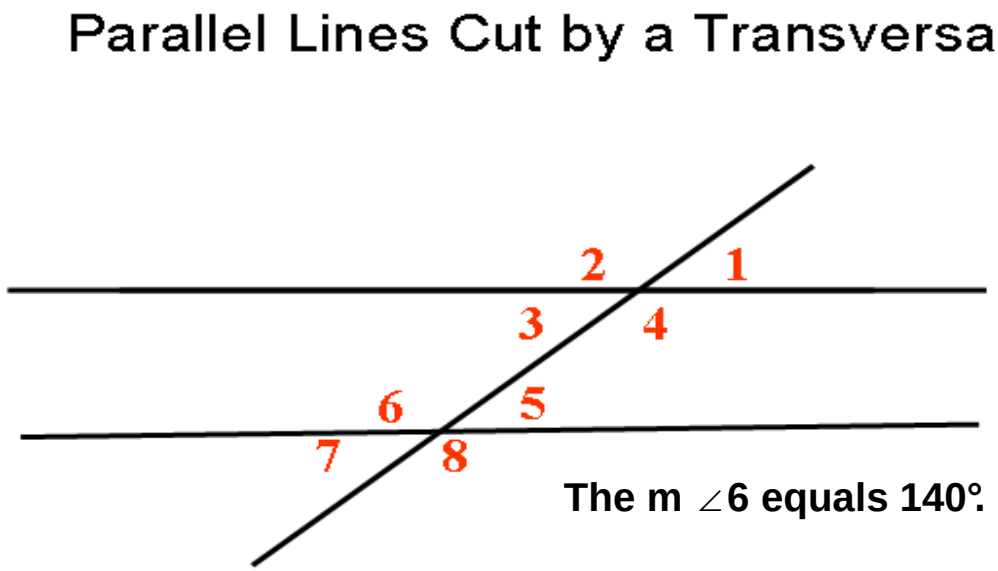
What are the measures of the other numbered angles?

Work and writing space:

Investigating Lines Cut by Transversals
Test Your Knowledge

Directions: Show your work and explain your reasoning using complete sentences.

3.



What are the measures of the other numbered angles?

Find the measure of each angle.

Work and writing space:

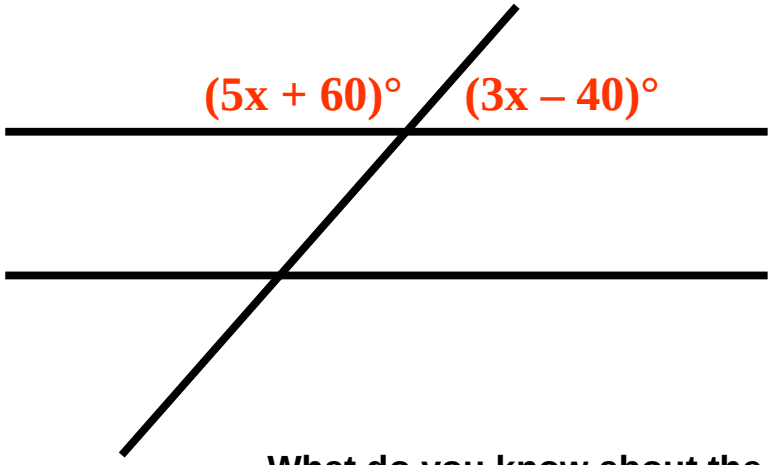
Investigating Lines Cut by Transversals
Test Your Knowledge

Directions: Show your work and explain your reasoning using complete sentences.

4.

Parallel Lines Cut by a Transversal

What are the measures of the angles below?



What do you know about the angles?
Write the equation.
Solve for x.
Find the measure of each angle.

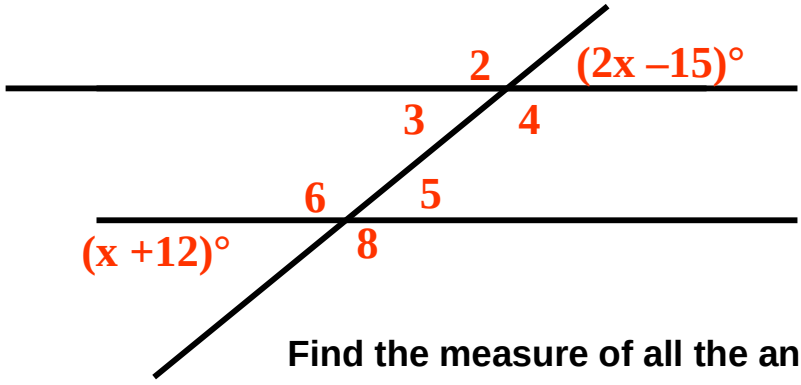
Work and writing space:

Investigating Lines Cut by Transversals
Test Your Knowledge

Directions: Show your work and explain your reasoning using complete sentences.

5.

Parallel Lines Cut by a Transversal



Find the measure of all the angles.

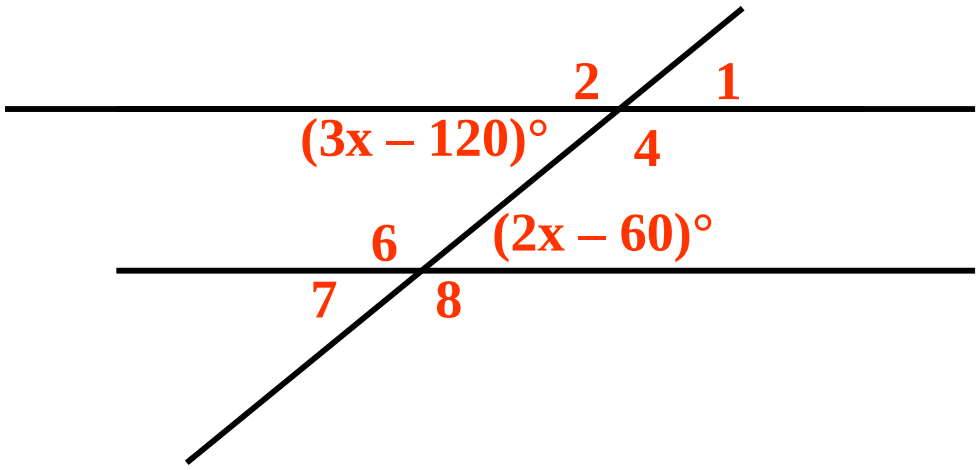
Work and writing space:

Investigating Lines Cut by Transversals
Test Your Knowledge

Directions: Show your work and explain your reasoning using complete sentences.

6.

Parallel Lines Cut by a Transversal



Find the measure of all angles.

Work and writing space: