Title:	Grade: 8	BIG Idea:
Pythagoras in the Real World	Author(s): Hope Phillips	Pythagorean Theorem

## Real-World Connection:

You don't have to work in the construction industry to understand this lesson! The Pythagorean Theorem is used widely in the world of construction. The mantra "Be Square or Repair" refers to the need for a building's foundation to be square, or so that the corners meet at right angles. A building is only as secure as the foundation upon which it sits. If a foundation is not square,

The basis of a foundation is a footing. The **footing** is the thing upon which the home rests. It is where the home meets the soil and the first member of the load-



bearing structural systems of the home.

Footings are almost always concrete. The footing is usually



formed by concrete poured into a trench.

Footings spread the load (weight) of a structure. There are "dead" and "live" weights. Dead weights are building materials. Live weights include people, furniture, rain, wind, and snow.

http://www.home-building-answers.com/footing.html (source for pictures)

Source for information below:

http://www.raisedfloorlivingpro.com/footings.shtml

Footing requirements are generally covered in the building code and sized in accordance with the bearing capacity of the soil and the weight of the building. In areas subject to seasonal frost, the bottom of the footing must be placed below the frost line to prevent damage to the footing and structure due to frost heave.

Source for information below: http://www.csgnetwork.com/foundationsquarecalc.html

The calculation is based on the Pythagorean Theorem. Reduced to simple construction terms, it says that the foundation length squared plus the foundation width squared equals the foundation diagonal distance (opposite corner to opposite corner) squared. It

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is also known in the construction industry as 3-4-5, meaning that if the length is 3 (squared = 9) and the width is 4 (squared = 16) then the diagonal is 5 (squared = 25). Verification is true in that 9 plus 16 is 25, of which the square root is 5. While protractors and T-Squares are very useful, the longer the distance for the measurement, the more accurate it is. (If you don't want to actually measure but can determine by a sting, both diagonals must be equal for a squared foundation.)

Just a fraction of a degree deviation from 90 degrees, over 20 feet or more, is a substantial error in causing walls to not be straight or square. Other problems include an improperly-fitting roof, walls that have cracks, and tile that must be cut at strange angles.

Source for information below:

http://wiki.answers.com/Q/How\_does\_a\_carpenter\_use\_Pythagorean\_Theorem#ixzz1r YnJFEYy

When laying out concrete footings for a new building, the Pythagorean Theorem is the most accurate method available for making square 90-degree angles. It is the same as the old 3 - 4 - 5 carpentry trick, only more precise, because the exact corners can be located.

Video Clip Photo Print Media (article, ad, etc.) Vodcast	Podcast Other
GPS Standards:	Objectives:
<ul> <li>M8G2: Students will understand and use the Pythagorean Theorem.</li> <li>a. Apply properties of right triangles, including the Pythagorean Theorem.</li> <li>b. Recognize and interpret the Pythagorean Theorem as a statement about areas of squares on the sides of a right triangle.</li> </ul>	<ol> <li>Students will use square tiles to demonstrate a proof of the Pythagorean Theorem.</li> <li>Students will determine the length of the hypotenuse given the length of two legs of a right triangle.</li> <li>Students will apply the Pythagorean Theorem to a real-world scenario.</li> </ol>

#### How Students will Experience the Connection: highlight in yellow all that apply

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#### Materials:

Calculator - one per student or student pair

Non-lined paper – one per student

Protractor - one per student

Ruler – one per student

Square tiles - 9 of one color; 16 of another color --- per student or student pair

Rope/cord/string – enough to equal the lengths of classroom walls plus the diagonal extending from corner to corner of the walls

Tape – number of rolls depends on how teacher proceeds with activity

Scissors - number of pairs depends on how teacher proceeds with activity

30-foot measuring tape – number depends on how teacher proceeds with activity

## Related Task:

Provide students with background information about footings, including the photographs. Emphasize the application of the Pythagorean Theorem in the real world. Ask students what they know about the theorem. Students may simply recite the symbolic representation,  $a^2 + b^2 = c^2$ .

Tell students they will be learning two important aspects of the Pythagorean Theorem in this lesson -- what the theorem *means* and where it can be applied in the real world.

Using a protractor, ask students to draw a right triangle on their paper with legs measuring 3 inches and 4 inches. Locate the drawing near the center of the paper. Display the following definition and equation on the board:

The Pythagorean Theorem that states that in a right triangle, the square of the length of the hypotenuse equals the sum of the squares of the lengths of the legs.  $a^2 + b^2 = c^2$ 

Ask students what "square of the length" means to them. Ask students, "If a number is squared, what 2-D figure will it make?" Answer: a square! So, what figures could we make on the legs and hypotenuse of the square? Answer: squares.

#### Tell students they will be making actual squares on the legs of the triangle and

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hypotenuse. Tell students that a "picture" of the definition and equation will make more sense to them. Pass out the square tiles to each student group.

Ask students the value of  $3^2$  and  $4^2$ . Ask students to build squares of 9 (3 x 3) and 16 (4 x 4) on the appropriate legs of their right triangles. Tell students that, according to the theorem, the sum of the squares of the lengths of the legs is equal to the square of the length of the hypotenuse. What, then, should be the sum of the square of the hypotenuse?

 $3^{2+}4^{2=}$ 9 + 16 = 25

So, the square of the hypotenuse should be a  $5 \times 5$  and composed of 25 squares.

Tell students to reassemble their  $3 \times 3$  square and  $4 \times 4$  square on the hypotenuse to form a  $5 \times 5$  square. With the two different tile colors, students will be able to see that the square of the hypotenuse is, indeed, the sum of the squares of the legs.



Once students are comfortable with the meaning of Pythagorean Theorem, tell them that they will be determining whether their classroom is "square."

\*Note: Any two adjacent walls may be used for this activity. It is the teacher's decision to do a whole-class demonstration or have small groups work together in multiple corners of a space – i.e. classroom, gym, cafeteria, etc.

String a line of rope measuring the length of one wall. Tape the ends to the wall. String another line of rope measuring the length of an adjacent wall. Tape the ends to the

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wall. Using the metal measuring tape, measure the length of both ropes.

Ask students to calculate the length of the hypotenuse for these two walls. Remind students that if the room is "square," the hypotenuse they will measure should match their calculations. Now string a line of rope connecting the corners of the adjacent walls. Discuss whether or not the actual measurement matched the predicted value.

Tell students that for very large building foundations, contractors would not want to measure, by hand, the length of two adjacent surfaces. That would be an inefficient use of time. Knowledge of the Pythagorean Theorem can make the measuring process much more efficient. Tell students that contractors use the *3-4-5 Rule*.

Ask students to recall the triangle they drew earlier. It, too, was a 3-4-5 triangle. Ask students how they could use this triangle to re-measure the classroom walls without having to string the long rope. The length of two adjacent walls doesn't matter as long as a length of 3 and 4 units can be measured. If 3 and 4 unit lengths can be measured, then the hypotenuse should be 5 units. This short-cut makes it much easier to determine "squareness".

# **EXTENTION:**

Discuss Pythagorean Triples. These are a set of three non-zero whole numbers (a, b, and c) so that  $a^2 + b^2 = c^2$ . There are infinitely many Pythagorean triples because one can multiply each original a, b, and c by the same value. For example, given a 3-4-5 triangle, multiplying by 2 results in a new triangle, 6-8-10. Given the 6-8-10 triangle, multiplying by 3.5 results in a new triangle, 21-28-35.

## Learn More:

- Pictures can be copied/pasted into a Word document or PointPoint slide



## Additional information on footings:

- The footing is the base of a structure and the first step in a foundation. It is a mass of concrete supporting the foundation of the house.
- The footing is the bottom part of the foundation and is usually made of concrete and reinforced with steel (rebar). The footing forms the base of the foundation and spreads the vertical loads from building.
  - $\circ$   $\,$  To transfer the live and dead loads of the building to the soil over a large

enough area so that neither the soil nor the building will move;

In areas where frost occurs, to prevent frost from moving the building

- **Dead loads** are the weight of the building materials and the soil surrounding the foundations.
- Live loads include the weight of people, furniture, snow, rain, and wind. Wind may be a vertical force downward, a horizontal force, or an uplift force

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- Houses may have spread footings that support the perimeter walls. These footings are wide pads that are continuous around the perimeter of the house. In some cases, the pads may be widened and/or thickened to accommodate concentrated loads from fireplaces, pilasters, etc.
- The concrete footing is the most important part of the house.
- The footing inspection is the most critical inspection that any building inspection department can make.

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