Title: INCOMPLETE TENNIS MATCH PROBLEM
PROBABILITY AND SIMULATION

Standards used:

Conditional Probability and the Rules of Probability

• Understand independence and conditional probability and use them to interpret data

• Use the rules of probability to compute probabilities of compound events in a uniform probability model

MCC9-12.S.CP.1 Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events ("or," "and," "not").

MCC9-12.S.CP.2 Understand that two events A and B are independent if the probability of A and B occurring together is the product of their probabilities, and use this characterization to determine if they are independent.★

MCC9-12.S.CP.3 Understand the conditional probability of A given B as P(A and B)/P(B), and interpret independence of A and B as saying that the conditional probability of A given B is the same as the probability of A, and the conditional probability of B given A is the same as the probability of B.

Using Probability to Make Decisions

• Calculate expected values and use them to solve problems

• Use probability to evaluate outcomes of decisions

MCC9-12.S.CP.6 Find the conditional probability of A given B as the fraction of B's outcomes that also belong to A, and interpret the answer in terms of the model.

MCC9-12.S.CP.7 Apply the Addition Rule, P(A or B) = P(A) + P(B) – P(A and B), and interpret the answer in terms of the model.

MCC9-12.S.CP.8 (+) Apply the general Multiplication Rule in a uniform probability model, P(A and B) = [P(A)]x[P(B|A)] =[P(B)]x[P(A|B)], and interpret the answer in terms of the model.

Problem: The plan is to play 5 games of tennis in which $64.00 goes to the winner. History has shown that the players are evenly matched. Jeff wins games 1 and 3 and Pat wins game 2. The score is Jeff 2 and Pat 1. Because of rain and travel, the match cannot be finished. How should the money be divided fairly? What questions do we need to ask before proceeding?

Use simulation to decide the likelihood of either Pat or Jeff winning.

ASSUMPTIONS: 5 games will be played
Players are evenly matched.
Games are independent.

- Suppose that only 2 games had been played.
- Suppose that only 1 game had been played.
- Suppose that we are going to play 1o games.

- Keep the original situation, but suppose that Jeff is twice as good as Pat.
  P(Pat wins)= 1/3 P(Jeff wins)= 2/3

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Real life: Your school's tennis team is in the state competition and the finals are interrupted by weather. How can you decide equitably who would have won?

OR

Use Wimbledon as the stakes and say that the players from out of the country cannot return to the US.

Solution: For the Simulation portion before you begin to work the theoretical probability you could use anything that you can set up as ½ (or equal possibilities of win/loss) such as, flip a coin, use dice (even/odd or 1,2,3 and 4,5,6), calculator random integer function (use 0,1 etc.).

A tree diagram works best. The tree will allow students to see what has already happened and that is past history and does not need to be taken into account. The 3 possibilities that could have already happened are JJP, JPJ, or PJJ. These are done deals so we do not need to look at the probability of that.
Now… we have 2 more games to play. There are 3 possibilities of things that could still happen.
1 – P(J wins 4^{th} game) = ½ (if this happens the tournament is over because that is the 3^{rd} win for Jack.
2 – P(P and then J) = ¼ (if that happens Jack still wins)
3 – P(P and P) = ¼ (this is the only way that Pat could win)

SO

P(Jack wins tournament) = ¾
P(Pat wins tournament) = ¼

So Jack $3/4^{th}$ of \$64.00 = \$48.00
And Pat gets $1/4^{th}$ of \$64.00 = \$16.00

For the remainder of the scenarios use the same reasoning and recreate the tree diagram and discuss when to complete the various branches when someone has won the tournament.

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