Title: Building with Tiles	**Grade:** 8	**BIG Idea:** Linear Patterns & Algebraic Thinking

CCGPS Standards Addressed:

MCC8.F.2 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).

Note: Given the list of multiple representations above, this lesson focuses on the *pictorial* representation as a precursor to the representations listed.

MCC8.F.5 Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear.) Sketch a graph that exhibits the qualitative features of a function that has been described verbally.

Mathematical Practices Emphasized:

1. Make sense of problems and persevere in solving them.

2. Reason abstractly and quantitatively.

3. Construct viable arguments and critique the reasoning of others.

5. Use appropriate tools strategically.

7. Look for and make use of structure.

Learning Goals:

1. Students will represent a growing pattern pictorially, as a table, algebraically, and graphically.

2. Students will determine multiple, equivalent expressions that convey the growth of the pattern.

2. Students will determine a simplified expression for the "nth" step of a growing pattern.

3. Students will determine a function that represents the growing pattern.

Materials:

Teacher:
Handout projected on screen/SmartBoard
Electronic color tiles (or color tile manipulatives if interactive whiteboard not available)
Online graphing application (i.e., NCTM's Core Tools; Geogebra; etc.)

Per student:
Handout Colored Pencils Color Tiles Material to graph function in 1st quadrant

Teacher Notes:

This problem is a growing patterns exercise. Students will represent the pattern as a picture, in a table, and as symbols. Ask students to make observations about what is happening from Step 1 to Step 2 and Step 2 to Step 3, etc. (i.e., the figure is increasing in size in a constant way). What is changing? What remains the same?

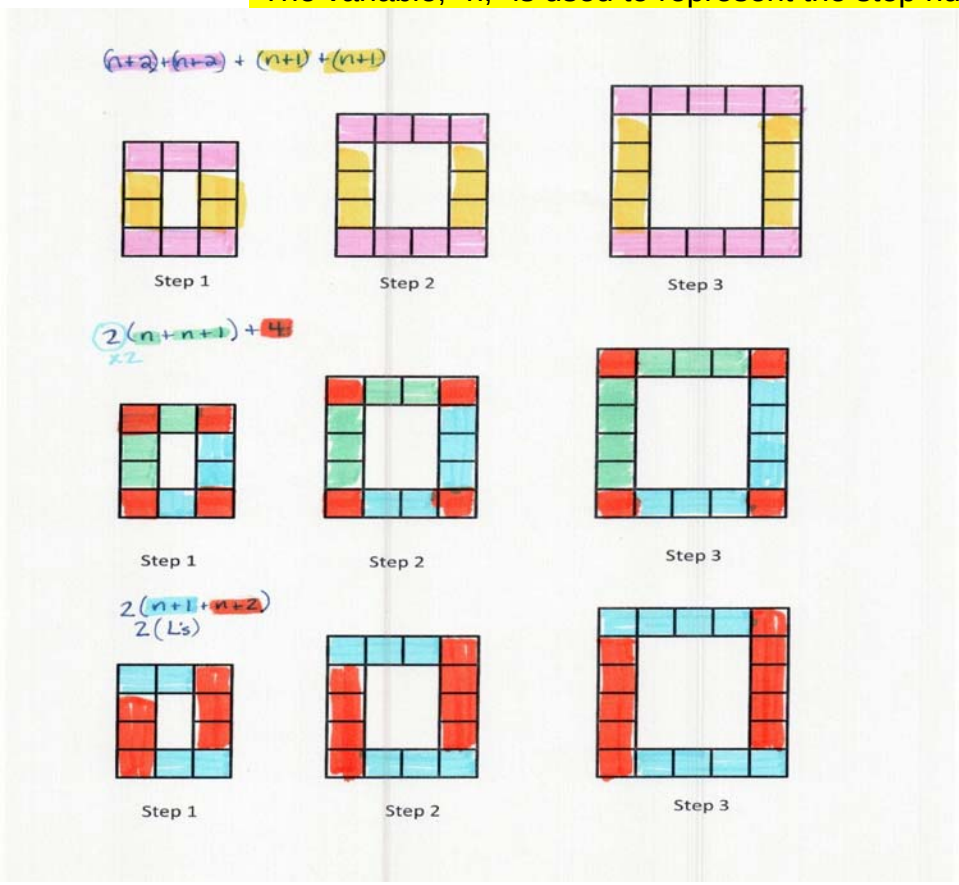
Ask students to represent the 4th and 5th steps using Color Tiles and/or by drawing the figures in order to determine the total number of tiles in each of these steps. These pictorial representations should reflect *how* the figure is growing and how this growth is constant from one figure to another. There are numerous ways to think about this *how* question.

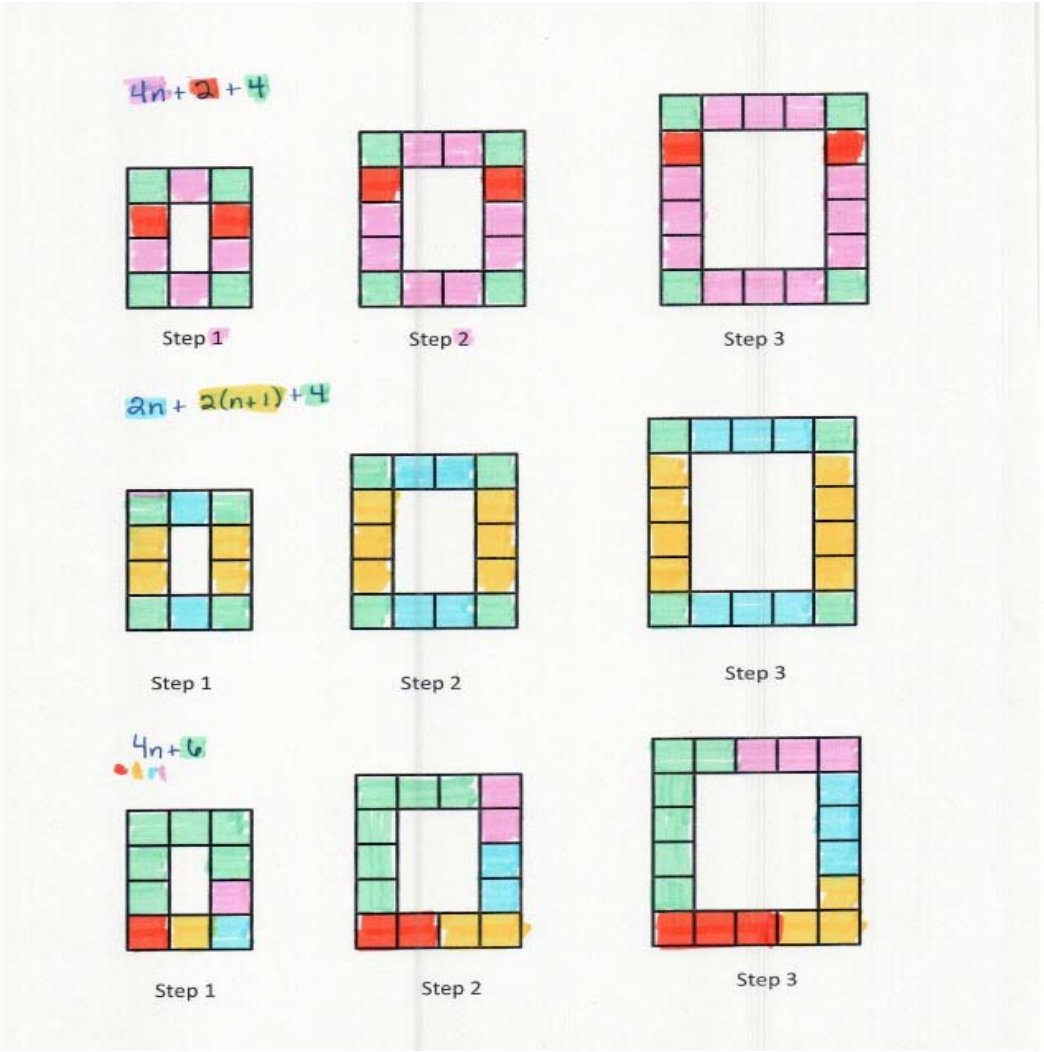
Note: When students are finished color-coding their representations, the discussion must lead students to develop expressions to represent their color coding. The link between the "step number" and the total number of tiles must be discussed. The color-coded representations below use the variable, "n," in place of the "step number."

After students have finished their work, have a class discussion that focuses on the many ways students have represented the growth.

Below is a series of diagrams representing 12 possible expressions for the same growing pattern. The growing patterns have been color-coded to reflect how the expressions could be developed by students. These 12 expressions are not, however, an exhaustive list.

*The variable, "n," is used to represent the step number.





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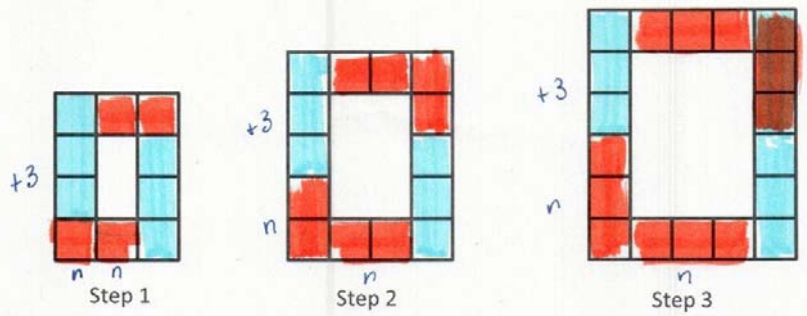
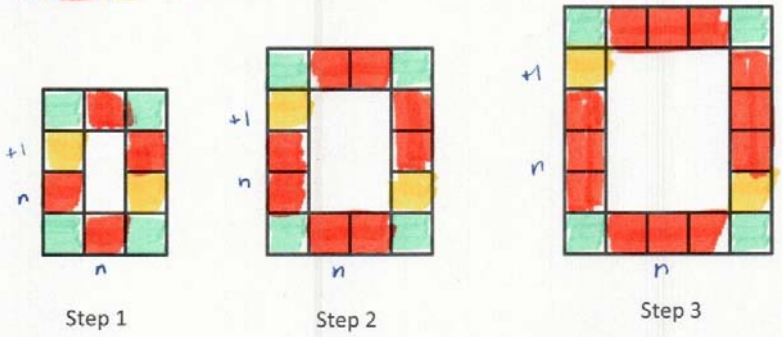
$2(n + n + 3)$

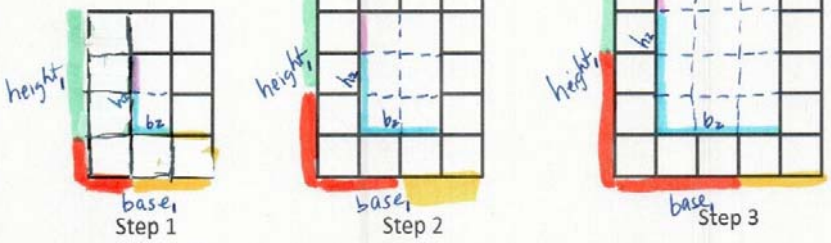


$2(2n+1) + 4$



$(n+2)(n+3) - n(n+1)$

$b_1 \times h_1$ $b_2 h_2$



area of whole large rectangle − area of inner rectangle

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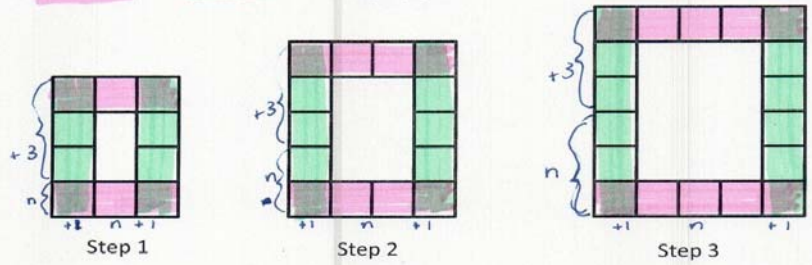
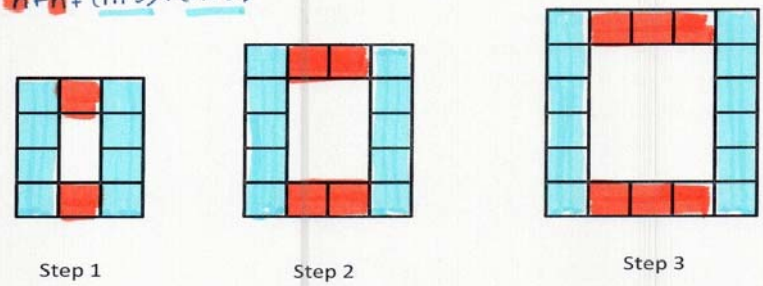
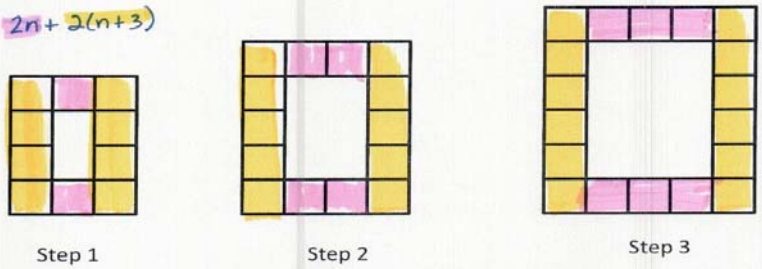
$2(n+2) + 2(n+3) - 4$ take out double counted corners



$n+n+(n+3)+(n+3)$



$2n+2(n+3)$



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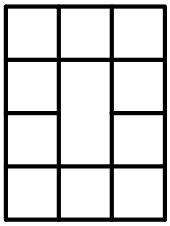
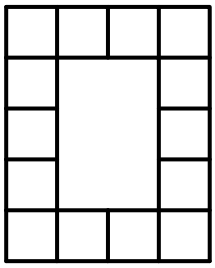
The focus of the next part of this lesson should be the "nth" step. Using the color pattern(s) they have discovered, ask students to determine how the 25^{th} step would look. Lead a discussion about how cumbersome it would be to draw or use tiles to represent this step. Ask students how they might discover a one-size-fits-all rule to determine the amount of tiles in *any* step number. That is, a rule for the *nth term*.

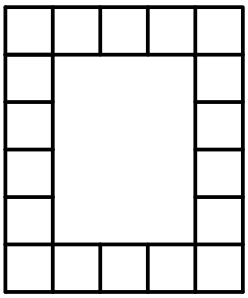
Directions for students: Write a formula for Step *n*. Color-code your picture to support the formula you have written.



Step 1



Step 2



Step 3

Each of the previous 12 expressions simplies to the expression 4n + 6. It is important that students realize that this simplified expression does not necessarily best show the way the pattern grows. It is just the algebraically- simplified expression.

To help bridge among the pictorial representations, the various algebraic expressions, and the function that can be written, ask students to create a table of values for this problem. Students will need to identify the two variables upon which they will be acting.

Step Number	Total Number of Tiles
1	10
2	14
3	18
n	4n + 6

Let "x" represent the step number and "y" the total number of tiles. Write an equation in function notation that expresses the total number of tiles in any step.

y = 4n + 6

It may be more meaningful, given the context of the problem, for students to express the function as

t = 4n + 6 where "t" is total tiles and "n" is the step number

Given the function t = 4n + 6, have students discuss what the graph would look like on a coordinate plane. Consider the following questions:

1. What depends on what? Does the number of tiles depend on the step number? Or, does the step number depend on the number of tiles? **Answer: The number of tiles depends on the step number. This informations lets us know how to label our axes. X axis: step number (independent). Y axis: total number of tiles (dependent).**

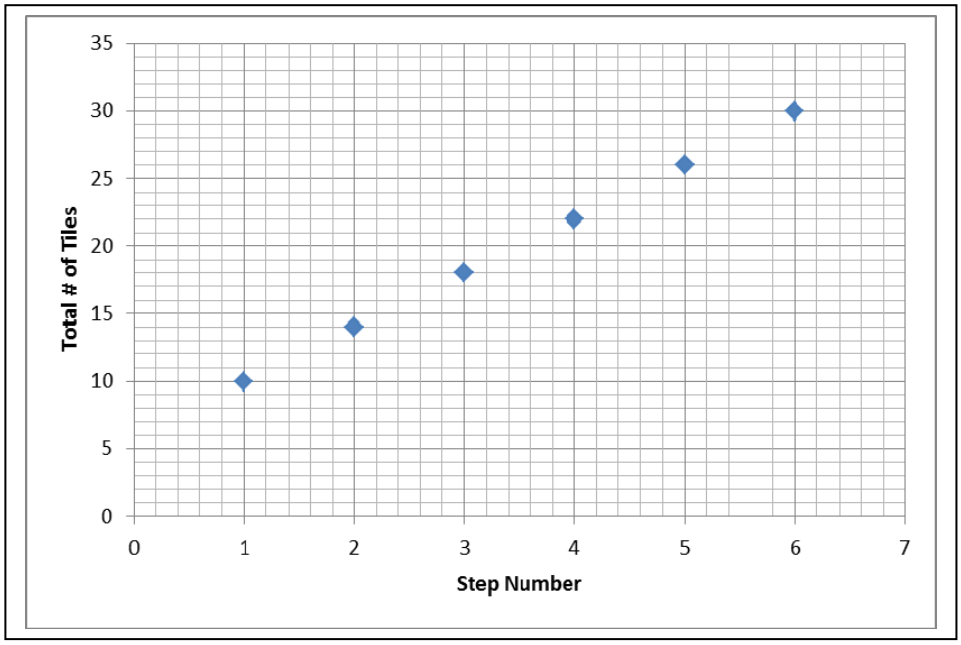
2. Will we need a four-quadrant coordinate plane? **Answer: No. There would be no such value as a negative step number on the "x" axis or a negative number of tiles.**

3. What kind of function do you expect the growing pattern to be? Why? **Answer: Linear. Because there is a constant rate of change from step to step.**

4. Will the graph of the function increase or decrease? Why? **Answer: Increase. As the step number increases, the resulting number of tiles also increases.**

5. After graphing the data, should we draw a line between the points? Why? **No. This is an example of discrete data. There is not a possibility of a partial step, such as Step 1 ½.**

Ask students to graph the function to verify (or alter) their answers. Below is the graph of the function.



Notice the absence of a y-intercept because, in this context, it doesn't make sense. There is not a "step 0." Also, notice that there is not a line. This is a discrete function, not a continuous function. Therefore, a line should not be drawn connecting the points because there is not a "step 1.5," for example.

Using the function you created, which step number would have 74 tiles?

$t = 4n + 6$

$74 = 4n + 6$

$74 – 6 = 4n + 6 – 6$

$68 = 4n$

$17 = n$

Step 17 would contain 74 tiles.

Using a graphing application (e.g., NCTM's Core Tools; Geogebra; Geometer's Sketchpad; TI-83; etc.), students can extend (i.e., extrapolate) the graph of the function to determine whether "17" is a reasonable answer for the question above.