Title: Ye Ole Volume	**Grade: 8**	**BIG Idea:** Exponents

CCGPS Standards Addressed:	**Learning Goals:**
MCC8.EE.3 Use numbers expressed in the form of a single digit times an integer power of 10 to estimate very large or very small quantities, and to express how many times as much one is than the other. **Mathematical Practices Emphasized:** 1. Make sense of problems and persevere in solving them. 4. Model with mathematics. 5. Use appropriate tools strategically.	• Students will express the old English system of measuring volume as powers of two. • Students will apply the rules of exponents to a real-world situation.

Materials:

Handout of units for measuring in Old England (attach) – one per student or student group

Per student group:

Cup measure
Tablespoon measure
Teaspoon measure

*Use these materials to help familiarize students with typical units of liquid and dry measures in the customary system.

Teacher Notes:

To the right is a list of the units for measuring volume in early England. Some of the measurements are commonly used for liquid (e.g., gallons) and others are commonly used for dry goods (e.g., grain).

Ask students what they notice numerically about all of the conversion factors. One important fact is that each measurement is twice (2x) the previous measurement. Beginning with the smallest unit, 1 mouthful, double it to get 1 jigger. Now double 1 jigger and get 1 jackpot. This pattern continues throughout the table. The measurements double from the smallest, the mouthful, to the largest, the tun.
Ask students if this reminds them of any mathematical phenomenon – something related to powers and exponents. They might even recognize that the measurements act like place value.

2 mouthfuls	=	1 jigger
2 jiggers	=	1 jackpot
2 jackpots	=	1 gill
2 gills	=	1 cup
2 cups	=	1 pint
2 pints	=	1 quart
2 quarts	=	1 pottle
2 pottles	=	1 gallon
2 gallons	=	1 peck
2 pecks	=	1 half-bushel
2 half-bushels	=	1 bushel
2 bushels	=	1 cask
2 casks	=	1 barrel
2 barrels	=	1 hogshead
2 hogsheads	=	1 pipe
2 pipes	=	1 tun

For example,

10 ones = 10
10 tens = 100
10 hundreds = 1000
10 thousands = 10,000
And so on…

Using exponents, we can re-write the above relationships.

$10=10^1$
$100 =10^2$
$1000 = 10^3$
And so on…

Ask students to think about the English measurements and the doubling relationship.

How might we use exponents to express these relationships – that is, impose a mathematical model (Standard for Mathematical Practice #4) on a real-world phenomenon.

The given measurements produce the following relationships expressed as ratios:

$$\frac{2\ mouthfuls}{1\ jigger} \times \frac{2\ jiggers}{1\ jackpot} \times \frac{2\ jackpots}{1\ gill} \times \frac{2\ gills}{1\ cup} \times \frac{2\ cups}{1\ pint} \times \frac{2\ pints}{1\ quart} \times \frac{2\ quarts}{1\ pottle} \times \frac{2\ pottles}{1\ gallon} =\# \frac{2^8\ mouthfuls}{gallon}$$

Using dimensional analysis, the "like" units, in the numerators and denominators, divide to one (i.e., jiggers and jiggers; jackpots and jackpots; etc…) The result is 2^8 mouthfuls for

every gallon, or 256 $\frac{mouthfuls}{gallon}$

Ask students how they would represent the relationships above using exponents.

Typically, students think of starting exponent notation with n^1. Ask students which of these ratios represents n^1.

Remember that the relationships among these measurements begins with and grows from the ratio $\frac{2\ mouthfuls}{1\ jigger}$. We can represent “mouthfuls to jiggers” as 2^1 because there are 2 mouthfuls per 1 jigger.

From this starting ratio, students should recognize that each subsequent measure is double the previous measure. This doubling results in exponential growth. There are 4 mouthfuls per jackpot. 8 mouthfuls per gill. 16 mouthfuls per cup. And so on…

The ratios below can be represented using exponential notation.

$\frac{2\ mouthfuls}{1\ jigger}$	2^1
$\frac{2\ mouthfuls}{1\ jigger} \times \frac{2\ jiggers}{1\ jackpot}$	2^2
$\frac{2\ mouthfuls}{1\ jigger} \times \frac{2\ jiggers}{1\ jackpot} \times \frac{2\ jackpots}{1\ gill}$	2^3
$\frac{2\ mouthfuls}{1\ jigger} \times \frac{2\ jiggers}{1\ jackpot} \times \frac{2\ jackpots}{1\ gill} \times \frac{2\ gills}{1\ cup}$	2^4
$\frac{2\ mouthfuls}{1\ jigger} \times \frac{2\ jiggers}{1\ jackpot} \times \frac{2\ jackpots}{1\ gill} \times \frac{2\ gills}{1\ cup} \times \frac{2\ cups}{1\ pint}$	2^5
$\frac{2\ mouthfuls}{1\ jigger} \times \frac{2\ jiggers}{1\ jackpot} \times \frac{2\ jackpots}{1\ gill} \times \frac{2\ gills}{1\ cup} \times \frac{2\ cups}{1\ pint} \times \frac{2\ pints}{1\ quart}$	2^6
$\frac{2\ mouthfuls}{1\ jigger} \times \frac{2\ jiggers}{1\ jackpot} \times \frac{2\ jackpots}{1\ gill} \times \frac{2\ gills}{1\ cup} \times \frac{2\ cups}{1\ pint} \times \frac{2\ pints}{1\ quart} \times \frac{2\ quarts}{1\ pottle}$	2^7
$\frac{2\ mouthfuls}{1\ jigger} \times \frac{2\ jiggers}{1\ jackpot} \times \frac{2\ jackpots}{1\ gill} \times \frac{2\ gills}{1\ cup} \times \frac{2\ cups}{1\ pint} \times \frac{2\ pints}{1\ quart} \times \frac{2\ quarts}{1\ pottle} \times \frac{2\ pottles}{1\ gallon}$	2^8

We have established that the mouthful is the *unit measurement*, or the one on which all the others are based. And, the smallest unit in our relationships above is 2^1, or 2. However, what if we wanted to determine a unit based on **1**, **not 2**, amount of some measure. How could we do this?

Using our smallest measure, divide 2 mouthfuls in half. 2 mouthfuls becomes 1 mouthful. Now, our unit measurement is based on *1 unit*, not 2 units. Dividing by 2/2, versus 2/1, maintains equivalent ratios.

$$\frac{2 \text{ mouthfuls}}{1 \text{ jigger}} + \frac{2}{2} = \frac{1 \text{ mouthful}}{\frac{1}{2}\text{jigger}}$$

With this change all ratio relationships are now based on $\mathbf{2^0}$ or **1** mouthful. That 1 mouthful is equivalent to ½ jigger is not important at this point.

***Note:** The determination of the mouthful as the *unit* is arbitrary. It was selected because it is the smallest unit. Intuitively, it makes sense to build all the other measurements from the smallest measurement. However, if we wanted to make the cup the unit (a very recognized measure to students), we simply need to re-envision the exponential notation.

When expressing exponential notation in any base (i.e., base 10, 2, etc…), it is customary to work from smaller to larger unit. Think about the organization of the base-ten system.

Ten thousand	10,000	**1 x 10^4**
thousand	1000	**1 x 10^3**
hundred	100	**1 x 10^2**
ten	10	**1 x 10^1**
one	**1**	**1 x 10^0**
tenth	$\frac{1}{10}$	**1 x 10^{-1}**
hundredth	$\frac{1}{100}$	**1 x 10^{-2}**
thousandth	$\frac{1}{1,000}$	**1 x 10^{-3}**
Ten-thousandth	$\frac{1}{10,000}$	**1 x 10^{-4}**

Now compare the base-ten system to the early English measurement system. Notice that the cup has been selected as the *unit*. This selection is based on the United States customary units where all measurements (i.e., pint; quart; and gallon) are based on the cup.

gallon	**16**	$\mathbf{2^{4}}$	**2 x (2 x 2 x 2 x 1)**
pottle	**8**	$\mathbf{2^{3}}$	**2 x (2 x 2 x 1)**
quart	**4**	$\mathbf{2^{2}}$	**2 x (2 x 1)**
pint	**2**	$\mathbf{2^{1}}$	**2 x (1)**
cup	**1**	$\mathbf{2^{0}}$	**1**
gill	$\frac{1}{2}$	$\mathbf{2^{-1}}$	$\frac{1}{2}\times(1)$
jackpot	$\frac{1}{4}$	$\mathbf{2^{-2}}$	$\frac{1}{2}\times(\frac{1}{2}\times 1)$
jigger	$\frac{1}{8}$	$\mathbf{2^{-3}}$	$\frac{1}{2}\times(\frac{1}{2}\times\frac{1}{2}\times 1)$
mouthful	$\frac{1}{16}$	$\mathbf{2^{-4}}$	$\frac{1}{2}\times(\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\times 1)$

2 mouthfuls	=	1 jigger
2 jiggers	=	1 jackpot
2 jackpots	=	1 gill
2 gills	=	1 cup
2 cups	=	1 pint
2 pints	=	1 quart
2 quarts	=	1 pottle
2 pottles	=	1 gallon
2 gallons	=	1 peck
2 pecks	=	1 half-bushel
2 half-bushels	=	1 bushel
2 bushels	=	1 cask
2 casks	=	1 barrel
2 barrels	=	1 hogshead
2 hogsheads	=	1 pipe
2 pipes	=	1 tun

Double 1 **pottle**. 2 pottles is equivalent to 1 gallon. There are 16 cups per gallon.	2 x (2 x 2 x 2 x 1)
Double 1 **quart**. 2 quarts is equivalent to 1 pottle. There are 8 cups per pottle.	2 x (2 x 2 x 1)
Double 1 **pint**. 2 pints is equivalent to 1 quart. There are 4 cups per quart.	2 x (2 x 1)
Double 1 **cup**. There are 2 cups per pint.	2 x (1)
Beginning point -- 1 cup, or 2^{0}	**1**
Halve 1 **cup**. ½ cup is equivalent to 1 gill.	$\frac{1}{2}\times(1)$
Halve 1 **gill**. ½ gill is equivalent to a 1 jackpot.	$\frac{1}{2}\times(\frac{1}{2}\times 1)$
Halve 1 **jackpot**. ½ jackpot is equivalent to 1 jigger.	$\frac{1}{2}\times(\frac{1}{2}\times\frac{1}{2}\times 1)$
Halve 1 **jigger**. ½ jigger is equivalent to 1 mouthful.	$\frac{1}{2}\times(\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\times 1)$

Halving

In America we don't use mouthfuls as a unit of measure. However, 1/16 of a cup is a popular unit of measure. We just need to equate 1 mouthful, or 1/16 of a cup, with this popular unit of measure.

Ask students what smaller units of liquid or dry measure they are familiar with (e.g., tablespoons and teaspoons). Present the idea that we could see if a mouthful was equal to one of these measures or a fractional part of them (e.g., ½ teaspoon; ¼ teaspoon; ½ tablespoon, etc.). How we could find which of these measures is equivalent to 1/16 of a cup, the mouthful?

Using previous knowledge or an internet search, we need to know a couple of conversions about tablespoons and teaspoons.

$\frac{3\ TSP}{1\ TBS}$ and $\frac{48\ TSP}{1\ cup}$

From this information, we can determine the number of TBS per cup –

$$\frac{1\ TBS}{3\ TSP} x \frac{48\ TSP}{1\ cup} = \frac{16\ TBS}{1\ cup}$$

There are 16 TBS per cup. Look at the reciprocal of the ratio above $\frac{1\ cup}{16\ TBS}$

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There are two ways to interpret this ratio.

- 1 cup is equivalent to 16 tablespoons.
- 1 tablespoon is equivalent to 1/16 of a cup.

This value, **1/16 of a cup, is equal to the one mouthful**! We now have a useful conversion for the mouthful, and our journey is complete.

References:

Table of measurements from –

Reconceptualizing Mathematics
Judith Sowder, Larry Sowder, Susan Nickerson
ISBN 1-4292-2427-4
p. 535