Title: Proportional Reasoning with Manipulatives & Tables	**Grade(s):** 6	**BIG Idea:** Proportional Reasoning

CCGPS Standards Addressed:

MCC6.RP.1 Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities.

MCC6.RP.2 Understand the concept of a unit rate a/b associated with a ratio a:b with b ≠ 0 (b not equal to zero), and use rate language in the context of a ratio relationship.

MCC6.RP.3 Use ratio and rate reasoning to solve real-world and mathematical problems, e.g. by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.

Mathematical Practices Emphasized:

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
4. Model with mathematics. Students can model problem situations symbolically (tables, expressions or equations), visually (graphs or diagrams) and contextually to form real-world connections.
5. Use appropriate tools strategically. Students choose appropriate models for a given situation, including tables, expressions or equations, tape diagrams, number line models, etc.
7. Look for and make use of structure.

Learning Goals:

1. Students will use manipulatives to represent proportional relationships.
2. Students will understand that a ratio is a comparison of two quantities and that this relationship remains fixed.
3. Students will create a tabular representation of the work they do with manipulatives.
4. Students will represent proportional relationships in multiple ways.

Materials:
Teacher:

PowerPoint of questions

Student Groups (2 to 3 students per group):

1” square Color Tiles (or other material that can be used as counters – two different colors)
Paper/pencil for each student

Teacher Notes:

Introduce the PowerPoint by telling students that the focus of this lesson will be *mathematical relationships*. Ask students what they know about relationships in general (e.g., between sunlight and plant growth; studying and classroom success; outdoor heat and air conditioner use). Narrow the focus to mathematical relationships (e.g., division and subtraction; multiplication and addition; the number of zeros in a product when multiplying by a power of 10).

Show the first PowerPoint slide, and ask students what relationship is presented in the problem (i.e., the relationship between the number of purple and non-purple candies in a bag of candy). Working in small groups using color tiles, ask students to represent this initial relationship of 3 purple candies for every 5 non-purple candies. (Tip: Use only one color to represent the non-purple tiles). The representation should look like the following:

3 **purple**

5 non-purple

Ask students what their next step(s) should be in order to determine how many non-purple candies will be in a bag with 27 purple candies. Emphasize the vocabulary "for every" so that students will be cued to "bring in" (a reference to actually moving tiles as a group) 5 non-purple tiles for every set of 3 purple tiles. The act of bringing in a certain number of non-purple tiles for every group of purple tiles reinforces the "for every" relationship that defines a ratio. Remind students that the relationship between the two colors will not change no matter how many times they bring in sets of 3 purple and 5 non-purple tiles.

Special care has been made to avoid the use of the words "add more tiles" so that students will not focus on additive reasoning. The focus, rather, should be on the multiplicative relationship that is occurring. For example, the teacher could pose questions like the following: "If we **double** 3 purple candies and **double** 5 non-purple candies, what will result? If we **triple** 3 purple candies and **triple** 5 non-purple candies, what will occur? If we bring in **4 times as many** purple candies, how many non-purple candies must we bring in?"

As students use the tiles to build the relationships, ask them to record their pictorial work using a table. See sample below –

Purple	**3**	**6**	**9**	**12**	**15**	**18**	**21**	**24**	**27**
Non-purple	**5**	**10**	**15**	**20**	**25**	**30**	**35**	**40**	**45**

Some students will need to complete the entire table as shown above. However, some students will recognize the multiplicative relationships of doubling, tripling, etc…
Each group of students may have a slightly different "aha" moment, and, as a result, their tables may look different.

For example, after finding the 9:15 relationship, some students will see that 27 is three times as much as 9, and 45 is three times as much as 15. Multiplication by 3 (3 x 9 and 3 x 15) will result in the correct number of purple to non-purple candies.

Regardless of how far they had to extend their table and/or manipulative representations, ask students how the final results (27:45) are equivalent to the beginning ratio relationship of 3:5. While there is 9 times as many purple and non-purple candies, the relationship is equivalent to the 3:5. Simplified, the ratio 27:45 is 3:5. Notice this relationship is not part-whole; it is part-part.

The second problem is identical in structure to the first. However, because 5 and 4 are such small numbers, it would be more efficient for students to use manipulatives to establish the relationship and then use a table to finish the problem. (Answer: 48 purple candies for every 60 non-purple candies, or a 4:5 ratio). There is 12 times as many candies in the final ratio as in the original ratio, 48:60 and 4:5, respectively.

The first two questions represented a part-to-part ratio; the third problem introduces a part-to-whole ratio. Students must think about the missing part, or the non-orange candies to solve this problem. Using manipulatives the students must have three groups of tiles, instead of two. The physical representation will help students see that this problem is different in structure than the previous two. Students will likely pick out 2 orange tiles and have to consider how they will represent the total of 9 candies. The missing link is the 7 non-orange tiles. Now, they can represent a total of 9 candies (tiles) with 2 orange and the rest non-orange.

This time, building the table by doubling, tripling, etc. is an inefficient way to determine the total amount of candies. Encourage students to begin thinking in large "chunks" versus incrementally.

Orange	**2**	**4**	**6**	**8**	**10**	**12**	**14**	**16**	**18**	**20**
Non-orange	**7**	**14**	**21**	**28**	**35**	**42**	**49**	**56**	**63**	**70**
Total Candy in Bag	**9**	**18**	**27**	**36**	**45**	**54**	**63**	**72**	**81**	**90**

Consider asking, "How many times as many orange candies do we need than we have currently?" If we have 2 orange candies and need 20 orange candies, we need 10 times as much as we have.

This is a one-step approach ("chunking") to solving the problem using multiplicative reasoning. If we need 10 times as many orange candies, we will need 10 times as many non-orange candies. In total, the entire amount of candy we need will be 10 times as much as we have currently.

Ask students why 2:7 and 2:9 are the equivalent of 20:70 and 20:90, respectively. Make certain students know that these ratios are equivalent because of the Multiplicative Identity Property of 1.

$\frac{2}{7} \times \frac{10}{10} = \frac{20}{70}$ Orange to Non-Orange

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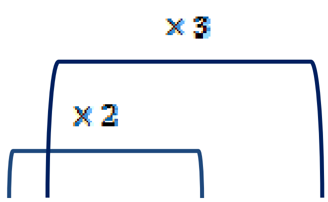
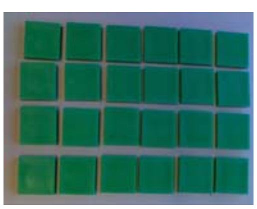
$\frac{2}{9} \times \frac{10}{10} = \frac{20}{90}$ Orange to Total

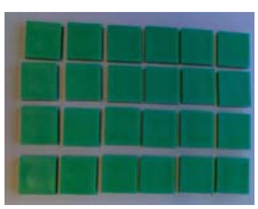
The fourth problem is also a part-to-whole ratio. This time, however, the multiplier is not a whole number. Students will notice this by using the manipulatives and/or setting up a table of values (see below). By doubling and tripling, one undershoots (i.e., 48) or overshoots (i.e., 72) the goal of determining the number of cookies for a 35-person party. Correspondingly, doubling and tripling the people results in 28 (below) and 42 (above), respectively. However, by knowing these incorrect values, we can determine that we need *more than* 2 times as many cookies but *less than* 3 times as many. The number of cookies will be between 48 and 72.

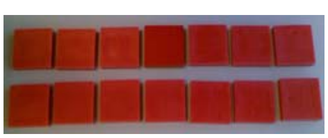


Cookies	24	48	72
People	14	28	42

We can better analyze this situation using tiles. One set of cookies can be represented by 24 tiles. We know we need 2 sets of tiles (i.e., 2 sets of 24, or 48 tiles). A third set of 24 tiles will be too much because this corresponds to 42 people (i.e., 3 times as many as 14).


1 group of cookies
24

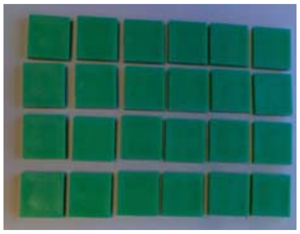

1 group of cookies
24


1 group of people
14


1 group of people
14

Ask students, “Given 28 people, how many are we lacking to have 35 people?” We are 7 short of our goal of 35. Ask students, “How does 7 compare to the group size of 14? 7 is one-half of 14. So, we only need *one-half* of a group of 14, and, therefore, only *one-half* of a group of 24 cookies. ½ of 24 is12.

Our finished representation with manipulatives will look like this –



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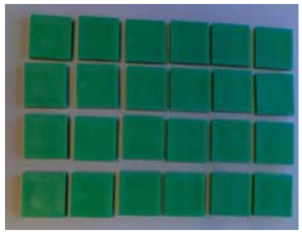

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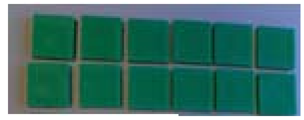

12

14

14

7

Using multiplication, instead of addition, to reinforce the multiplicative reasoning that is foundational to proportional reasoning, we represent symbolically the above as –

$$2\frac{1}{2}\times 24 = \ 2(24)+\frac{1}{2}(24)=48+12=60$$ **COOKIES**

$$2\frac{1}{2}\times 14 = 2(14)+\frac{1}{2}(14)=28+7=35$$ **PEOPLE**

Cookies	60	12
People	35	7

The ratio of cookies to people is **60**:35 or **12**:7.