CCGPS Mathematics

Standards for Mathematical Practice

1) Make sense of problems and persevere in solving them.

2) Reason abstractly and quantitatively.

3) Construct viable arguments and critique the reasoning of others.

4) Model with mathematics.

5) Use appropriate tools strategically.

6) Attend to precision.

7) Look for and make use of structure.

8) Look for and express regularity in repeated reasoning.

Mathematical Practices are listed with each grade's mathematical content standards to reflect the need to connect the mathematical practices to mathematical content in instruction.

The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices rest on important "processes and proficiencies" with longstanding importance in mathematics education. The first of these are the NCTM process standards of problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical proficiency specified in the National Research Council's report *Adding It Up*: adaptive reasoning, strategic competence, conceptual understanding (comprehension of mathematical concepts, operations and relations), procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently and appropriately), and productive disposition (habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy).

1 Make sense of problems and persevere in solving them.
High school students start to examine problems by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. By high school, students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. They check their answers to problems using different methods and continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

2 Reason abstractly and quantitatively.
High school students seek to make sense of quantities and their relationships in problem situations. They abstract a given situation and represent it symbolically, manipulate the representing symbols, and pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Students use quantitative reasoning to create coherent representations of the problem at hand; consider the units involved; attend to the meaning of quantities, not just how to compute them; and know and flexibly use different properties of operations and objects.

3 Construct viable arguments and critique the reasoning of others.
High school students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. High school students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. High school students learn to determine domains to which an argument applies, listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

4 Model with mathematics.

High school students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. High school students making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

5 Use appropriate tools strategically.
High school students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. High school students should be sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. They are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

6 Attend to precision.
High school students try to communicate precisely to others by using clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

7 Look for and make use of structure.
By high school, students look closely to discern a pattern or structure. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y. High school students use these patterns to create equivalent expressions, factor and solve equations, and compose functions, and transform figures.

8 Look for and express regularity in repeated reasoning.
High school students notice if calculations are repeated, and look both for general methods and for shortcuts. Noticing the regularity in the way terms cancel when expanding $(x - 1)(x + 1)$, $(x - 1)(x^2 + x + 1)$, and $(x - 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, derive formulas or make generalizations, high school students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Connecting the Standards for Mathematical Practice to the Standards for Mathematical Content
The Standards for Mathematical Practice describe ways in which developing student practitioners of the discipline of mathematics increasingly ought to engage with the subject matter as they grow in mathematical maturity and expertise throughout the elementary, middle and high school years. Designers of curricula, assessments, and professional development should all attend to the need to connect the mathematical practices to mathematical content in mathematics instruction.

The Standards for Mathematical Content are a balanced combination of procedure and understanding. Expectations that begin with the word "understand" are often especially good opportunities to connect the practices to the content. Students who lack understanding of a topic may rely on procedures too heavily. Without a flexible base from which to work, they may be less likely to consider analogous problems, represent problems coherently, justify conclusions, apply the mathematics to practical situations, use technology mindfully to work with the mathematics, explain the mathematics accurately to other students, step back for an overview, or deviate from a known procedure to find a shortcut. In short, a lack of understanding effectively prevents a student from engaging in the mathematical practices.

In this respect, those content standards which set an expectation of understanding are potential "points of intersection" between the Standards for Mathematical Content and the Standards for Mathematical Practice. These points of intersection are intended to be weighted toward central and generative concepts in the school mathematics curriculum that most merit the time, resources, innovative energies, and focus necessary to qualitatively improve the curriculum, instruction, assessment, professional development, and student achievement in mathematics.

TASKS

The following tasks represent the level of depth, rigor, and complexity expected of all Analytic Geometry students. These tasks, or tasks of similar depth and rigor, should be used to demonstrate evidence of learning. It is important that all elements of a task be addressed throughout the learning process so that students understand what is expected of them. While some tasks are identified as a performance task, they may also be used for teaching and learning (learning/scaffolding task).

Scaffolding Task	Tasks that build up to the learning task.
Learning Task	Constructing understanding through deep/rich contextualized problem solving tasks.
Practice Task	Tasks that provide students opportunities to practice skills and concepts.
Performance Task	Tasks which may be a formative or summative assessment that checks for student understanding/misunderstanding and or progress toward the standard/learning goals at different points during a unit of instruction.
Culminating Task	Designed to require students to use several concepts learned during the unit to answer a new or unique situation. Allows students to give evidence of their own understanding toward the mastery of the standard and requires them to extend their chain of mathematical reasoning.
Formative Assessment Lesson (FAL)	Lessons that support teachers in formative assessment which both reveal and develop students' understanding of key mathematical ideas and applications. These lessons enable teachers and students to monitor in more detail their progress towards the targets of the standards.

Task Name	**Task Type** *Grouping Strategy*	**Content Addressed**
Similarity in the Coordinate Plane	Scaffolding Task *Individual/Partner Task*	Dilating a figure in a coordinate plane Enlarge or reduce a geometric figure using a given scale factor
Similar Triangles	Practice Task *Individual/Partner Task*	Discover the relationships that exist between similar figures using the scale factors, length ratios, and area ratios
Shadow Math	Scaffolding Task *Individual/Partner Task*	Determine missing side lengths of similar figures
Proving Similar Triangles	Scaffolding Task *Individual/Partner Task*	Identify Similar Triangles Use similarity theorems to prove that two triangles are similar
Pythagorean Theorem using Triangle Similarity	Constructing Task *Partner/Small Group Task*	Prove the Pythagorean Theorem using triangle similarity
Lunch Lines	Performance Task *Individual/Partner Task*	Prove theorems about lines and angles of parallel lines cut by one or more transversals
Triangle Proportionality Theorem	Performance Task *Individual/Partner Task*	Properties of the ratio of segments of parallel lines cut by a transversal
Challenges from Ancient Greece	Constructing Task *Partner/Small Group Task*	Perform geometric constructions using Euclidean tools: copying line segments, copying an angle, bisecting a segment, and bisecting an angle
Constructing Parallel and Perpendicular Lines	Constructing Task *Partner/Small Group Task*	Perform geometric constructions using Euclidean tools: constructing parallel and perpendicular lines
Constructions Inscribed in a Circle	Constructing Task *Partner/Small Group Task*	Perform geometric constructions using Euclidean tools: constructing polygons inscribed in a circle
Proving two Triangles are Congruent	Constructing Task *Partner/Small Group Task*	Use the definition of congruence, based on rigid motion, to develop and explain the triangle congruence criteria: ASA, SSS, and SAS, AAS, and HL
Triangle Proofs	Performance Task *Individual/Partner Task*	Prove theorems pertaining to triangles
Centers of a Triangle	Constructing Task *Partner/Small Group Task*	Prove the medians of a triangle meet at a point Bisect a segment and an angle Construct perpendicular lines, including the perpendicular bisector of a line segment

Constructing with Diagonals	Constructing Task *Partner/Small Group Task*	Prove theorems about parallelograms
Proving Quadrilaterals in the Coordinate Plane	Constructing Task *Partner/Small Group Task*	Prove theorems about parallelograms
Company Logo	Culminating Task *Individual/Partner Task*	Recognize congruence of plane figures and verify through rigid motions Prove congruence using geometric theorems

Task Name	Task Type ***Grouping Strategy***	Content Addressed
Horizons	Scaffolding Task *Individual/Partner Task*	Pythagorean Theorem
Eratosthenes Finds the Circumference of the Earth	Enrichment Learning Task *Small Group/Whole Group Task*	Triangles in a Circular Context
Discovering Special Triangles	Learning Task *Small Group Task*	45-45-90 and 30-60-90 Triangles
Finding Right Triangles in Your Environment	Learning Task *Partner/Small Group Task*	Applications of Right Triangles
Create Your Own Triangles	Learning Task *Individual/Partner Task*	Complementary Relationships
Discovering Trigonometric Ratio Relationships	Learning Task *Individual/Small Group Task*	Trigonometric Ratios
Find That Side or Angle	Learning Task *Partner/Group Task*	Inverses of trigonometric functions
Clyde's Construction Crew	Culminating Task/Assessment *Partner/Small Group Task*	Review of Unit Standards

Culminating Task: Company Logo

Adapted from Common Core-Aligned Task with Instruction Supports: Silicon Valley Mathematics Initiative, SCALE, New York City DOE, 2011

Mathematical goals

- Mathematicians recognize congruence of plane figures and are able to prove congruence using geometric theorems.
- Congruence of plane figures can be verified through rigid motions.
- Congruence can be used to solve problems and prove geometric relationships.

Common Core State Standards

MCC9-12.G.SRT.5 Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.

MCC9-12.G.CO.6 Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.

MCC9-12.G.CO.7 Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.

MCC9-12.G.CO.9 Prove theorems about lines and angles. Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.

MCC9-12.G.CO.10 Prove theorems about triangles. Theorems include: measures of interior angles of a triangle sum to 180 degrees; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.

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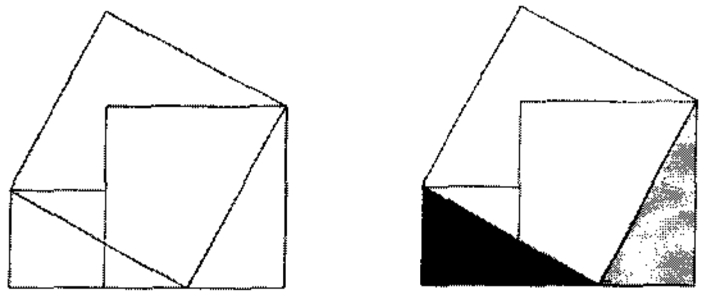
A company has designed a new logo using overlapping squares.



1. How many squares do you see in the logo?
   Describe where you see the squares.

2. The logo designer colored two triangles in the logo.
   How are the two triangles related?
   Justify your answer.

3. What are the relationships between the sizes of the squares in the original logo? Explain your findings.

Activity 7: Similar Triangles: An Outdoor Experiment

The Greek mathematician Thales, who lived in the 6th century BC, is generally acknowledged to be the first person to think of using similar triangles to calculate distances and heights that might be difficult to measure directly. Purportedly he used the method described below to calculate the height of the Great Pyramid, although this is difficult to confirm due to the annoying practice at the time of giving credit for any particular discovery to a man with a general reputation for wisdom.

Fig. 18

Regardless of whether Thales actually determined the height of the Great Pyramid or not, his method is sound. Let's use it to calculate the height of another famous Egyptian creation: the Great Sphinx (see Fig. 18). Carved out of the knoll of a rock around 2500 BC, this huge sculpture has the pharaoh Khafre's facial features atop the body of a lion.

Since most cultures don't like people clambering all over their burial monuments, Thales would most likely not have been permitted to climb to the top of the Sphinx and drop a tape measure. However, he would have been allowed to use the following method to find the height.

First, cut a straight 5 foot stick. Hold it upright and measure its shadow—say, 9 feet (see Fig. 19). Now walk to the base of the Sphinx and measure its shadow—118.8 feet. Since the sun is in the same place, the two shadow triangles must be similar.

$$\frac{5}{9} = \frac{x}{118.8}$$

$$9x = 594$$

therefore $x = 66$

Thus the Sphinx is about 66 feet tall.

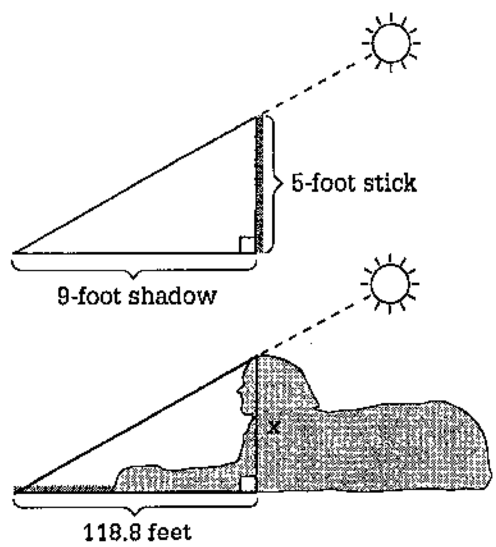


Fig. 19

EXPERIMENT

Goal:

Every morning the flag is raised in front of the school. What is its highest point? Determine the distance from the ground to the top of the flag on the flagpole (not to the top of the flagpole itself).

Materials:

Each pair of students should have:

- a yardstick (or at least a ruler)
- a 10 foot piece of string
- a calculator

Teacher should have:

- a long tape measure (check with the physical education department to borrow the one they use at track meets)
- a large safety pin

Procedure:

1. Height of yardstick is ______ feet, ______ inches. In decimal form this is ______ feet.
2. One student holds the yardstick perpendicular to the ground, holding end of string at its base. Partner walks to the end of its shadow and pulls the string taut. Measure the string. The yardstick's shadow is ________ feet, ________ inches. In decimal form this is ________ feet.
3. The height of the flag is x feet.
4. Teacher selects a pair of students (the first ones done) to measure the flagpole's shadow with the long tape measure (don't forget to only measure to the part of the shadow that shows the top of the flag). The distance is ________ feet, ________ inches.
In decimal form this is ________ feet.

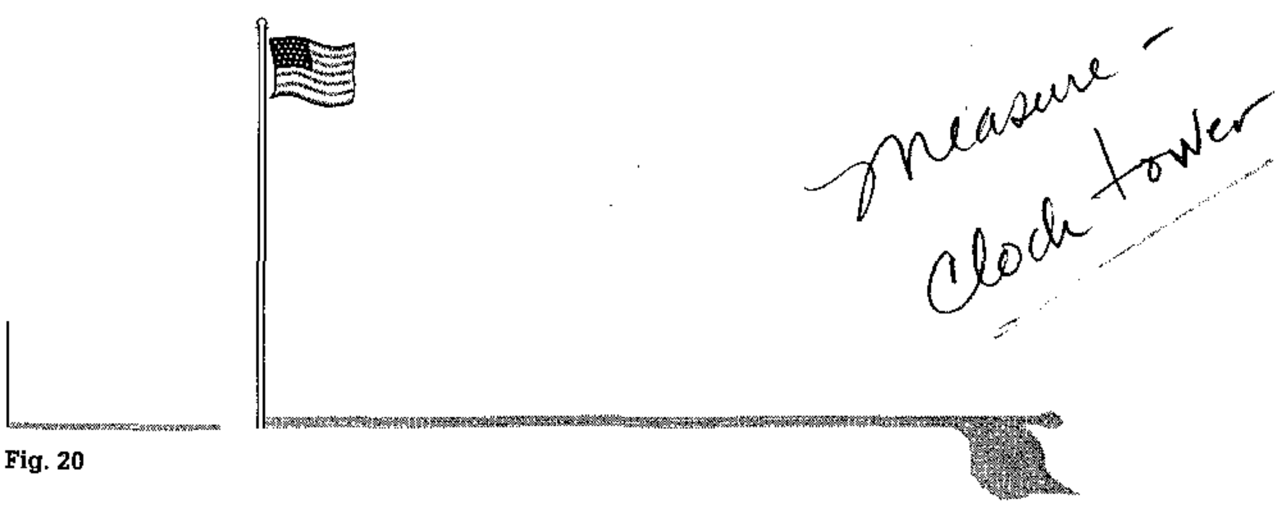


Fig. 20

Calculate:

Using similar triangles, find the height to the top of the flag. Your answer: ________ feet.

Data Analysis:

Gather an answer from each team. Discard any "outliers": answers that are wildly different from the rest of the group's. Average the rest of the scores. The average height is ________ feet.

Verification:

Select the team of students whose answer was closest to the class average to perform the verification procedure:

1. Let the flag down from the pole.
2. Pin the end of the tape measure to the top of the flag. Hoist the flag to the top, letting the tape out as you go. Hold the tape taut against the ground and measure: _________ feet, _________ inches. In decimal form this is _________ feet.

Error Analysis:

Percentage-wise, how close was the actual distance to the class average? ____________________

Write a paragraph to account for any discrepancies. ____________________________________

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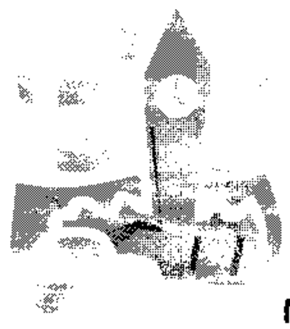
CSU Clock Tower



Make a Theodolite with your group. Use the card stock copy of a compass, weight, tape, string, and straw provided.

Take your theodolite and a large tape measure to the clock tower to measure the angle of inclination and the distance to the tower from the person measuring the angle.

Return to the classroom and draw a diagram of the clock tower and the measurements that you have taken. Use these measurements to calculate the height of the clock tower. Please show all work.

Now – Each group will use this activity to design an additional activity that utilizes these measurements with the Trig Trainer to create an activity for your students. (Be ready to share your activity with the class!)

Theodolite or Protractor

Use this diagram to construct the theodolite described in Activity 17 on page 35.