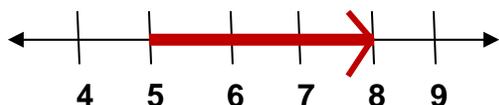
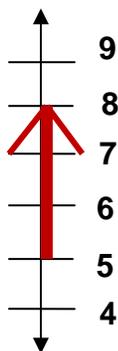


Title: Going the Distance -- Subtracting Integers	Grade: 7	BIG Idea: Subtraction of positive and negative integers
CCGPS Standards Addressed: MCC7.NS.1 Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal and vertical number line diagram. Standards for Mathematical Practice 2. Reason abstractly and quantitatively. 5. Use appropriate tools strategically. 8. Look for and express regularity in repeated reasoning.	Learning Goals: 1. Students will represent subtraction of integers using vertical number lines. 2. Students will interpret subtraction as “how many more or how many less”? 3. Students will develop the rule for subtracting integers based on repeated reasoning across numerous problems.	
Materials: For Teacher: Vertical number line Variety of subtraction of integer problems Per Student: Vertical number line handout		
Teacher Notes: In elementary school, most students are taught subtraction of whole numbers based on a <i>difference</i> or <i>takeaway</i> model. That is, given a problem like $8 - 5$, a common scenario is, “I have 8 objects. I take away 5. How many are left?” A scenario that is posed with much less frequency is, “Jack has 8 objects. Jill has 5. How many more objects does Jack have than Jill?” Generically, this question asks one to consider, “How much more is 8 than 5?” Modeled on a number line, “How much bigger is 8 than 5?” is actually a question of <i>distance</i> . The distance model is an appropriate way to understand subtraction of integers; The takeaway model is not.		



Movement is to the **right** or **positive** direction



Movement is **upward** or in the **positive** direction

In the examples above, 8 is 3 more than 5. Note that either a vertical or horizontal number line is appropriate. However, students may think of *distance* more readily using a vertical number line.

The distance model of subtraction can be generalized as the following:

How much _____ is _____ than _____?
 (more or less)
 (bigger or smaller)

Until they are comfortable with subtracting integers directly, students should vocalize and/or write this question using the numbers of the problem.

Below are the four types of integer subtraction problems students will encounter.

1. **5 – 8 Interpretation:** How much less is 5 than 8?

Using a number line, students should begin on 8 and move three down to 5. Because 5 is three less than 8, we note this difference with a negative sign. Also, when using a number line, students can notice that the movement is downward (e.g., vertical number line) or to the left (e.g., horizontal number line).

*Note: From this point, all references will be to vertical number lines.

2. **-5 – (-8) Interpretation:** How much more is -5 than -8?

Using a number line, students should begin at -8 and move up to -5. Because -5 is three more than -8, we note this difference as positive.

3. **-5 – 8 Interpretation:** How much less is -5 than 8?

Using a number line, students should begin at 8 and move down (vertical number line) to -5. Here zero may be used as a benchmark. Note that from 8 to 0 is a distance of 8. From 0 to -5 is a distance of 5. In total, that is a distance of 13. Because we are moving down, the resulting difference is negative, or -13.

4. **5 – (-8) Interpretation:** How much more is 5 than -8?

Using a number line, students should begin a -8. Here zero may be used as a benchmark. Note that from -8 to 0 is a distance of 8. From 0 to 5 is a distance of 5. In total, that is a distance of 13. Because we are moving in an upward direction, the resulting difference is positive, or 13.

The order of the problems above is not mandatory but, instead, gives students the opportunity to compare the processes for determining the differences and examine the resulting differences.

Generalizing based on patterns

Have students work multiple problems like the ones above. When you feel they possess conceptual understanding, ask students to examine all of their differences. (Our sample of problems is limited, but, in class there would be many more.) Students should be asked to examine the differences to see if there are any connections to integer addition, which they would have already learned.

1. $5 - 8 = -3$
2. $-5 - (-8) = 3$
3. $-5 - 8 = -13$
4. $5 - (-8) = 13$

If students need prompting, ask them to change all subtraction signs to addition. Then, ask them what they would need to do to the second integer, in each problem, to obtain the given answers. The second integer is replaced with “m” to indicate a missing number.

1. $5 + m = -3$ With the second number originally being 8, what would I have to do to 8 to make this number sentence true?
2. $-5 + x = 3$ With the second number originally being -8, what would I have to do to -8 to make this number sentence true?
3. $-5 + x = -13$ With the second number originally being 8, what would I have to do to 8 to make this number sentence true?
4. $5 + x = 13$ With the second number originally being -8, what would I have to do to -8 to make this number sentence true?

Depending on students' level of understanding of addition of integers, the previous addition scenarios may be difficult. The teacher may have to prompt students to change the second integer to its opposite. It is important that students set equal these problems (see below) to reinforce that subtraction of integers is equivalent to addition of the opposite.

1. $5 - 8 = 5 + -8$

2. $-5 - (-8) = -5 + 8$

3. $-5 - 8 = -13 = -5 + -8$

4. $5 - (-8) = 5 + 8$

Help students articulate that they are "adding the opposite of an integer" instead of relying on mathematically meaningless phrases, such as "chop-slice."