

# emcsquared: Exploring Changing Rates and Lines

<b>Title:</b> Exploring Changing Rates and Lines	<b>Grade: 6</b>  <b>Author(s):</b> Mary Lou Wilson Pearlie Jenkins Hope Phillips	<b>BIG Idea:</b> The constant, $k$ , determines the rate of change of a line.
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## Prior Knowledge Needed:

- How to graph coordinate pairs on a coordinate plane
- How to order fractions from least to greatest

## GPS Standards:

**M6A1** Students will understand the concept of ratio and use it to represent quantitative relationships.

### M6A2

**d.** Describe proportional relationships mathematically using  $y = kx$ , where  $k$  is the constant of proportionality.

### M6A2

**e.** Graph proportional relationships in the form  $y = kx$  and describe characteristics of the graphs.

### M6P1

**a.** Build new mathematical knowledge through problem solving (using appropriate technology).

## Objectives:

1 – Students will understand that the constant,  $k$ , determines the rate of change of a line.

2 – Students will understand that the graph of  $y = kx$  is always a straight line.

3 – Students will understand that the greater the constant,  $k$ , the greater the rate of change of the line.

4 – Students will use a computer applet to build their mathematical knowledge.

5 - Students will use a table, a graph, and the values of  $k$  to describe the rate of change.

## Materials:

- A number line (leave room to the left of 1 to suggest negative numbers and to the right of 1 to suggest values greater than 1) -- students may have a paper copy of a number line (see *Resources* section below) and/or the teacher may make a number line out of some kind of string/clothes line/etc. and display in the room; attach index cards using paper clips to display fractional and whole number values
- Cards marked 0,  $\frac{1}{2}$ ,  $\frac{4}{5}$ ,  $\frac{2}{3}$ ,  $\frac{2}{5}$ ,  $\frac{1}{5}$ ,  $\frac{3}{6}$ ,  $\frac{5}{2}$ , 3, 2, and 1 for classroom number line **OR** paper copy (see *Resources* section)
- Computers with internet access to applet (see *Resources* section)
- Colored pencils for students -- if students do not have access to the applet themselves

- graph paper

**Task:** (for answers to the task -- see *Resources* section below)

Warm-Up prior to beginning task: Using a number line\*, students should put the values (0,  $\frac{1}{2}$ ,  $\frac{4}{5}$ ,  $\frac{2}{3}$ ,  $\frac{2}{5}$ ,  $\frac{1}{5}$ ,  $\frac{3}{6}$ ,  $\frac{5}{2}$ , 3, 2, and 1) in order from least to greatest: Lead a general discussion about the correct ordering of the fractions. Highlight topics such as the following: size of fraction based on its numerator and denominator; sizes relative to  $\frac{1}{2}$ , 0, and 1; meaning of an improper fraction.

\*Students can become a number line by holding the cards and arranging themselves.

Use the computer applet (see *Resources* section) to graph each of the following equations on the same coordinate plane:

$$y = 1x$$

$$y = \frac{2}{3}x$$

$$y = \frac{1}{4}x$$

Place four lattice points (points with integer coordinates) on each line. Enter the coordinates of each data point in a table (provided on worksheet).

Think of each line as a staircase that one would climb -- from data point to data point. You may move from point to point in two ways -- up and down and left to right. You may **not** move diagonally.

How would you describe the way you moved from the first to the second point on the line  $y = 1x$ ? The second point to the third point? The third point to the fourth point? Are you climbing at the same rate for all the points? Explain.

Now try "climbing" from the first point you placed on the line to the third point. Compare this movement to the movement you took between adjacent points. What is different? What is the same?

Repeat the "climbing" process between points for the other two lines --  $y = \frac{2}{3}x$  and  $y = \frac{1}{4}x$ . Explain any similarities. Explain any differences.

Using the Geogebra graph tool applet, graph the following pairs of equations on new coordinate planes:

Coordinate plane #1:  $y = 1x$  and  $y = 3x$

Coordinate plane #2:  $y = \frac{2}{3}x$  and  $y = \frac{3}{2}x$

Coordinate plane #3:  $y = \frac{1}{6}x$  and  $y = \frac{1}{4}x$

What do you notice about the rates of change of each pair of lines?

What happens to the rate of change of a line as the constant,  $k$ , increases? Decreases?

What can you conclude about the constant,  $k$ , and its effect on the rate of change of the line?

**Description and Teacher Directions:**

**Teacher Commentary:**

What happens to the rate of change of a line as the coefficient of the variable,  $x$ , increases? Decreases?

Warm-Up: As students enter the classroom, hand out the ratio cards and have selected students place them on a number line. The remaining students will order the values on paper. A quick and low-cost way to create a life-sized number line is to use laundry cord and index cards attached with paper clips.

After the warm-up, introduce the lesson with the essential questions:  
What happens to the rate of change of a line as the constant increases?  
...decreases?

Have students open the applet, or the teacher may use the applet as a demonstration tool with students completing the work using graph paper and colored pencils. Students may work individually or in pairs.

This lesson is designed to show how the constant of proportionality,  $k$ , affects the rate of change of a line. By discussing the rate of change and its effect on a line, the emphasis is on the rate as a *constant ratio*. This approach to the constant,  $k$ , fits the sixth-grade GPS objectives for proportional reasoning and sets the stage for teaching the rate of change as slope in eighth grade. From sixth- to eighth-grade, these concepts are then unified. By using a tabular format, students will see that each successive ratio is equivalent, once simplified.

There is no need to refer to the *slope* of the lines at this point. The focus here is on *rate of change* and its connection to ratios. To help focus students' attention on the *rate of change*, consider asking, "How fast are the  $y$ -values growing?"

When using the applet: Select any of the lines to be graphed by clicking on it in order to change its properties (color, thickness, label). To accomplish this, right click after clicking on the line itself. A dialogue box should appear. Select "**Object Properties**" and change any of the suggested features to your liking. The entire graph may be moved by selecting the right-most icon at the top of the viewing window (entitled "**Move Graphics View**"). Being willing to explore the features of this applet using the menus at the top of the page will allow you to find numerous useful features.

Place three or four lattice points (points in the coordinate plane with integer coordinates) on each graphed line. To do this, click on the "**New Points**" button at the top of the page (second icon from the left). For ease of future calculations, place points with integer values only. Move the mouse slowly along the line to see the individual coordinates of each potential new point.

If students do not have access to computers, allow them to use a colored pencils to draw each pair of lines.

When students have entered the coordinates of several data

I teach this lesson before having the students graph manually. Students seem to enjoy using the applet. It allows students to quickly transfer the knowledge that I want them to gain from the task. They may graph manually as an extension.

points in a t-chart, ask them to simplify any ratio that is not already in lowest terms. Students should notice that the ratio ( $y/x$ ) of all data points -- for a single line -- is the same. They should also notice that these ratios are equivalent to the coefficient of the  $x$ -value in the equation of the line, or  $k$ .

The  $k$  value is also called the "rate of change" of a line and determines how steep it is. For example, the line  $y=x$  is steeper than the line  $y=1/4x$  because "1" is greater than "1/4". One multiplied times an  $x$ -value results in a larger number than 1/4 multiplied by the same  $x$ -values. Therefore, the resulting  $y$ -values of the  $1*x$  will be larger than  $1/4*x$ .

The value of the constant of proportionality,  $k$ , determines the steepness of the line.

### Resources:

*Georgia GPS Edition Coach: Standards-Based Instruction, Mathematics, Grade 6*

[http://emcsquared.pbworks.com/f/Graph\\_Tool.html](http://emcsquared.pbworks.com/f/Graph_Tool.html)

applet for activity

<http://donnayoung.org/f10/math-f/coordnt/dy-coordinate-grid-6.pdf>

Above site contains six, 10 x 10 grids

 [NumberLine\[Rates\].doc](#)

**paper copy of Warm Up for students; contains two copies of warm-up on one page**

**paper copy of task for students**

**answers to paper copy of task for students**

 [Assessment Item \[Exploring Changing Rates\].doc](#)

 [Assessment Item Answers \[Exploring Changing Rates\].doc](#)

[Rate of Change Assessment Items](#)