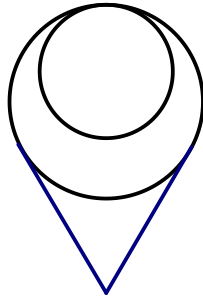


Earring Problem

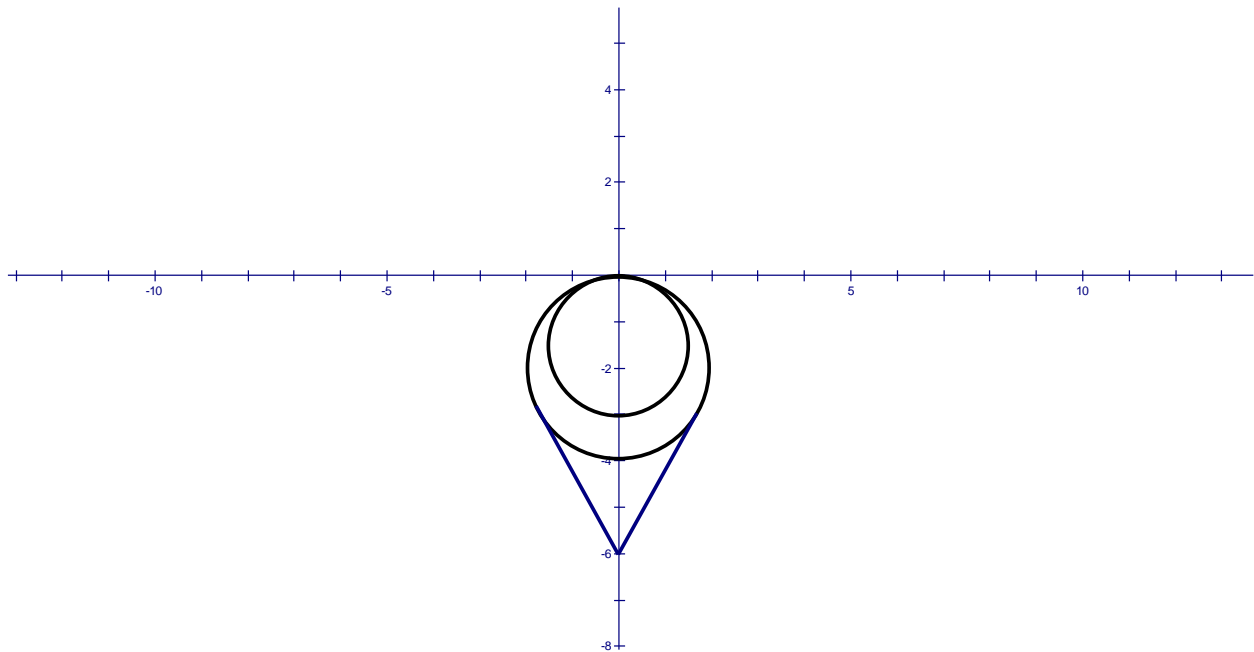
MCC9-12.G.GPE.1 Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.

- Translate between the geometric description and the equation for a circle

An earring has been lost! It was made of silver wire and a replica must be made like the existing old one. The earring has the shape of two circles that internally tangent with two tangents to the outer circle as shown top diagram below



A friend has a computer design system that can reproduce the earring if he can create the equations of the design on a coordinate grid. The bottom diagram shows a drawing of this earring related to the coordinate axis measured in centimeters. The circles touch at $(0, 0)$. The equation of the inner circle is $x^2 + y^2 + 3y = 0$. The outer circle intersects the y-axis at $(0, -4)$. The tangents meet the y-axis at $(0, -6)$. Find the total length of the silver wire required to make this earring.



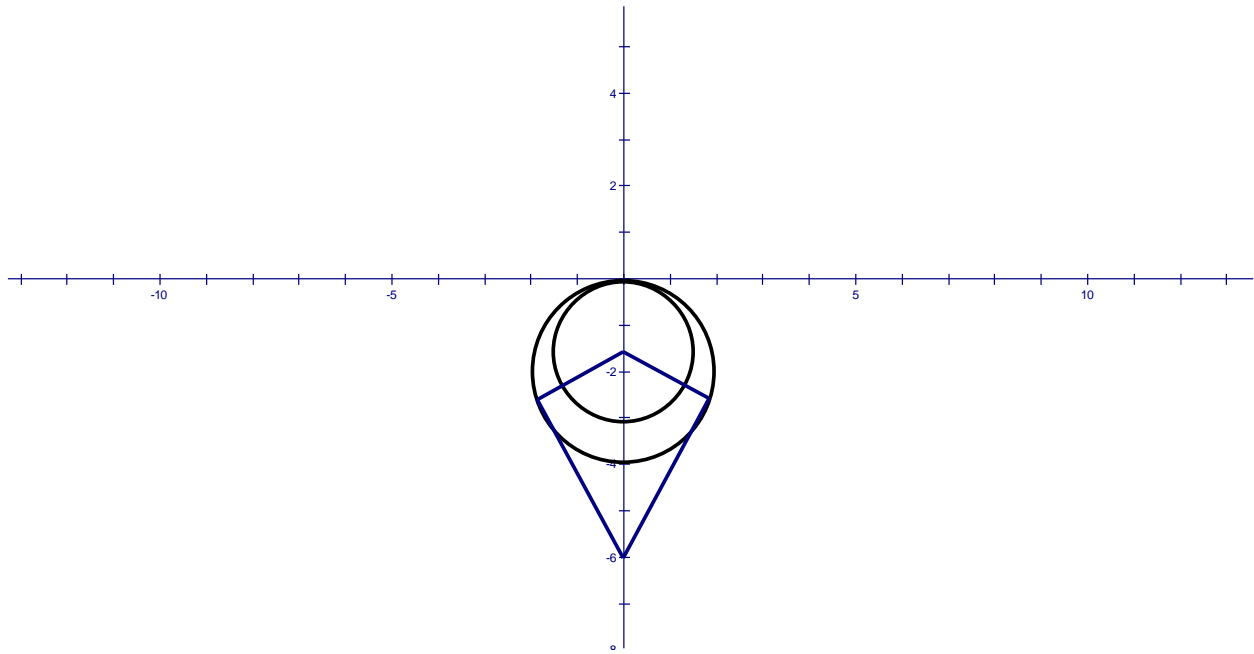
“Created by participants in Building Connections in High School Mathematics, a 2011 project of the Columbus Regional Mathematics Collaborative using Teacher Quality Funds.”

Solution:

Since the radius of inner circle is $3/2$ cm the circumference will be 3π cm.

The radius of the outer circle is 2 cm therefore the circumference will be 4π cm.

Since the lines are tangent to the outer circle we will have to use Pythagoras' theorem.



The hypotenuse of the triangle is 4 cm and the leg is 2 cm. Using Pythagoras' you find the length of the tangent line is $2\sqrt{3}$ cm. Since there are two tangent lines their lengths will be $4\sqrt{3}$ cm giving the entire length of line to be $7\pi + 4\sqrt{3}$ cm or 28.92 cm.