<table>
<tr>
<td>Title:

How Do You See It?

*Based on Counting Dots and Measuring Area: Rich Problems from Japan from the NCTM journal, Mathematics Teaching in the Middle School (November 2006)</td>
<td>Grade: 6</td>
<td>BIG Idea:

Algebraic Thinking & Expressions</td>
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<td>CCGPS Standards Addressed:

MCC6.EE.6 Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set.

Mathematical Practices Emphasized:

1. Make sense of problems and persevere in solving them.

2. Reason abstractly and quantitatively.

3. Construct viable arguments and critique the reasoning of others.

5. Use appropriate tools strategically.

7. Look for and make use of structure.</td>
<td colspan="2">Learning Goals:

1. Students will determine methods of counting a set of dots.

2. Students will generate numeric expressions to describe their counting methods.

3. Students will use variables to generate numeric expressions to describe their methods of counting the set of dots.

4. Students will understand the terms variable, constant, and coefficient.

*Learning Goals based on the presentation of the problem in the NCTM journal Mathematics Teaching in the Middle School article referenced below</td>
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<td colspan="3">Materials:

Two-color counters - 16 per student
Dots template handout – one per student
Colored pencils – up to four per student</td>
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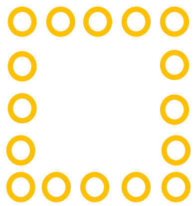
Teacher Notes:

After reading the *Mathematics Teaching in the Middle School* article (see *References* below) about the dot problem, this lesson emerged.

Introduce the problem by telling students they will use their mathematical creativity to generate as many ways as they can to count the dots in the formation below. They need to answer the basic question, "How many dots are there in the design?" Students should work alone or together, or a combination of the two.



Using colored pencils students should shade the circles to represent the manner in which they counted the dots (i.e., the shading will convey their counting strategies). Students should also provide a numerical expression that represents symbolically their methods of counting. Some students may prefer to work with the counters first, using the two different colors to represent their methods of counting. From there, colored pencils can be used to record their strategies.

Below are possible, but not an exhaustive list, of answers.

4(3) + 4
4 sets of 3 plus 4

2(5) + 2(3)
2 sets of 5 plus 2 sets of 3

2(9) – 2
2 sets of 9 minus 2

*Note: The corner pieces get counted twice in the 2(9), so they must be subtracted.

After students have time to generate multiple numeric expressions, have a whole class discussion where they share their work. Stress the importance of the symbolic representation matching the pictorial representation. Consider asking students to represent, pictorially, a given numeric expression and vice versa – represent, numerically, a given pictorial representation.

Keep a running count of how many representations are presented in order to impress upon students the value of different interpretations and multiple exit points (ways to represent the problem situation) for this problem.

According to the article, as a bridge to asking students to create an expression for the total number of dots in an "nth" or general case, ask them how to generate an expression for a larger number of dots. For example, a design with 25 or 100 dots. Ask students, "Would some/all of your methods work if a square had more than 16 dots, say 25 or 100?" Allow students to apply some/all of their counting strategies to squares with these larger numbers of dots.

When students are ready to move to the "nth" case, lead a discussion about why one would *want* to determine a general rule. Ask students if they would like to determine the number of dots in a square with 400 dots. Ask them if they could create a "shortcut" to determining the number of dots in a square design of any size, small or large. Mathematically speaking a "shortcut" is a *rule*.

To create a rule based on this expression, we must consider what parts are **constant** and what parts **vary**. These terms in bold print, *constant* and *vary*, lay the foundation for introducing the math vocabulary words *constant, coefficient,* and *variable*.

Let's analyze the expression 4(3) + 4 or 4 sets of 3 plus 4. The 4 red dots in the corners are fixed or constant, no matter the number of dots we have. The 4 sets of yellow dots are also fixed. That is, there will always be 4 sets of some amount of dots. The number that varies, however, is the amount of dots in these 4 sets.

In order to express the rule for 4(3) + 4, we will need to represent the number of yellow dots in each of the 4 set using a *variable.* A variable is a quantity that represents, or stands for, a number. Since the number of yellow dots in each set changes depending on the total number of dots, we cannot express this amount with a numeral like we can the number of dots in the corners, or 4. Select a letter for the variable that makes sense in context for the students. For example, "d" for dots.

So, the expression 4(3) + 4 can now be written as 4d + 4. The 4 representing the dots in the corners is a *constant* – 4 will always be 4. The 4 in front of the variable is a *co-efficient*. It serves as the *multiplier* in the expression. That is, it tells us how many sets of yellow dots we have.

The expression 2(5) + 2(3) is written as 2(d) + 2(d-2) because there will always be…

- 2 sets of dots on the left and right side (columns)
- The co-efficient "2" serves as the multiplier indicating how many columns of dots we have.
- 2 sets of dots on the top and bottom (rows) that are 2 less in quantity per row than each column
- The co-efficient "2" serves as the multiplier indicating how many rows of dots we have.

 The expression 2(9) – 2 is written as 2(d) – 2 because there will always be…

- 2 sets of "d" number of dots
- The co-efficient, 2, of the variable is a multiplier telling us how many sets of "d" dots we have
- The need to take away 2 dots because they were counted twice in the 2d term

So, to where does a lesson like this lead students? It is the pre-cursor to a growing patterns problem where students are given a series of pictures and their corresponding figure/step numbers. In these types of problems, students are asked to determine a general rule for the "nth" design, also. But in this case, the rule must correspond to the figure/step number. In the cases explained above, students are just introduced to the idea of *variable*, in general.

References:

*Dot problem from…

Counting Dots and Measuring Area: Rich Problems from Japan
Author: Blake Peterson
Mathematics Teaching in the Middle School
Vol.12; No. 4
November 2006