Title: Planning a Garden Space	**Grade:** 7 **Author(**: Hope Phillips	**BIG Idea:** Inverse Variation

Real-World Connection:

People enjoy attractively designed gardens, public parks and playgrounds, residential areas, college campuses, shopping centers, golf courses, and parkways. *Landscape architects* design these areas so they are not only functional but also beautiful and harmonious with the natural environment. They plan the location of buildings, roads, and walkways, and the arrangement of flowers, shrubs, and trees (Source: http://www.bls.gov/oco/ocos039.htm)

There are many brief online videos about landscape design from websites such as www.diynetwork.com and www.hgtv.com

How Students will Experience the Connection: highlight in yellow all that apply

Video Clip	Photo	Podcast
Print Media (article, ad, etc.)	Vodcast	Other

GPS Standards:

M7A3: Students will understand the relationship between two variables.

a. Plot points on a coordinate plane.
b. Represent, describe, and analyze relations from tables, graphs, and formulas.
c. Describe how change in one variable affects the other variable.
d. Describe patterns in the graphs of proportional relationships, both direct (y=kx) and inverse ($y = \frac{k}{x}$)

M7P3: Students will communicate mathematically.

Objectives:

1. Given a fixed area, students will determine all possible whole number dimensions of a rectangular flower bed.

2. Students will graph ordered pairs as (vertical length, horizontal length) on a coordinate plane.

3. Students will describe the characteristics of the graph of an inverse variation.

4. Students will write a rule in the form $y = \frac{k}{x}$ to describe the relationship between vertical and horizontal lengths of a fixed area, where *k* is the area.

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M7P4: Students will make connections among mathematically ideas and to other disciplines. **M7P5:** Students will represent mathematics in multiple ways.	

Materials:

Pencil – per student
1" square tiles – 24 per student
Handouts (2) – one each per student
Landscape Architects; The *Futures Channel* video*, Algebra in the Real World* or check www.thefutureschannel.com

Related Task:

Introduce a scenario like the following: A landscape architect's client expresses a desire for a tiled area in her backyard. The architect determines there is space for a 24-yd^2 rectangular space. As a part of her presentation to the client, the architect must show her client all of the different rectangles the arrangement of tiles could take. Show a short video clip of a backyard tiling installation.

Ask students what mathematical concepts they will be exploring when making rectangles with 24 tiles.

- What will change? (answer: the dimensions of length and width) The handout refers to the dimensions, length and width, as "vertical" and "horizontal" lengths because of the confusion between the terminology *length* and *width* and the ability to be interchange them.
- What will remain constant? (answer: the area will remain constant)
- In what units do we measure length? (answer: one-dimensional units, such as feet and yards; the square tiles are 1 inch x 1 inch)
- In what units do we measure area (Area = vertical length x horizontal length or 1 inch x 1 inch = 1 in^2 because area is a two-dimensional unit)

Using all 24 square tiles, ask students to determine all of the rectangular arrangements. Answers should be recorded on the handouts. Some students will use their tiles to find each pair of dimensions. Others will figure out that the dimensions of the rectangle are simply factors of 24.

Students will likely ask about whether to include both, for example, a 4 x 6 rectangle and a 6 x 4 rectangle. Mathematically, these are not different, just their orientation. However, in design, a backyard space may be able to accommodate one orientation but not the other. Discuss both the real-world application versus the mathematical application of orientation. These "repeat" dimensions will help

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students see more clearly the curved line formed on the coordinate plane upon graphing the data (i.e. 8 pieces of data, instead of 4).

Have a class discussion asking students how they will know when they have all of the rectangular dimensions. When the dimensions begin to repeat, with the vertical and horizontal lengths reversed, all of the rectangles will have been found. For example,

Vert. Length	Horiz. Length
1	24
2	12
3	8
4	6

The dimensions in the table on the left are the only arrangements of tiles that form unique rectangles. The next dimensions would be 6 (vertical) by 4 (horizontal). These dimensions are the reverse of the 4 x 6 rectangle. Similarly, the remaining dimensions would be 8 x 3; 12 x 2; and 24 x 1.

Many students will not list the dimensions in chronological order. This is perfectly acceptable. However, it *may* make determining when all the unique dimensions have been found more difficult to discern. Have a discussion about the benefits of systematically determining the dimensions.

Tell students they will be graphing their dimensions on a coordinate plane. They will graph the points as (vertical length, horizontal length) or (x, y). Tell students to label their table with “x” and “y” above the appropriate column.

Ask students what type of graph they believe the data points will form. If they suggest a line, ask students what direction they believe the line will fall on the coordinate plane. Frame questions like, “What happens to the horizontal length as the vertical length increases?” (Answer: As the vertical length increases, the horizontal length decreases) or “What happens to the vertical length as the horizontal length increases?” (Answer: As the horizontal length increases, the vertical length decreases.) If students do not suggest a curved line, they might intuit a line that falls from left to right.

Discuss the characteristics of the graph. The data points form a curved line that extends in both directions forever. The type of graph is a hyperbola. The data are discrete, not continuous. Since the lengths of the rectangles have been determined by measuring the length of a side of a one-inch tile, there are no partial tile lengths, such as 1/4 inch, 1 ½ inches, etc. This means that it would be inaccurate to draw a line connecting the dots *on the graph.* However, in the *real world*, not all tiles would be cut to whole-number dimensions. Fractional parts of feet would be possible.

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Discuss whether the graph will ever touch either axis. The answer is "no." Theoretically, the vertical and horizontal dimensions can continue to increase or decrease. Although, there would be a point where a dimension would become too large or too small to deal with realistically. Practically speaking, if one dimension were "0" there would be no rectangle at all.

This graph is an example of an inverse variation. In an inverse variation, the product of the "x" and "y" variables is a constant. The general form of the equation is y = $\frac{k}{x}$.

Ask students to use the graph for the purposes of prediction. Often students do not realize that graphs can be used to find data, as well. For example, ask students what the corresponding horizontal length of a rectangle with a vertical length of 5 would be. Have students use their graphs to answer before calculating the exact answer (Answer: $24 \div 5 = 4.8$). An answer of "almost five" is sufficient from reading the graph.

Lead students in a discussion about writing a rule for the graph. Have students analyze their table of values. Ask them what remains constant and what changes. An area of 24 is constant, while the vertical and horizontal dimensions are the variables, or values that can change. Ask students how area is calculated. Most will respond "length multiplied times width" or A = l x w. Remind students that they named these lengths "vertical" and "horizontal," so their formula should read **A = vertical x horizontal**.

Since the area is the constant value, we can substitute "A" for the variable *k* to represent the constant of proportionality. *Vertical* and *horizontal* can be substituted with "x" and "y." Now the formula should read *k* = *xy*. Ask students whether this mathematical statement is true for each data point. Since *k* represents the area of 24, the product of every "x" and "y" pair does, in fact, equal 24.

Therefore, substituting values in the general equation y = $\frac{k}{x}$,

Horizontal Length = $\frac{Area}{Vertical\ Length}$

Learn More:

- http://www.bls.gov/oco/ocos039.htm a look at the occupation of being a landscape architect

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Designing a Garden Space

Directions: Create dimensions of a rectangular garden with an area of 24 yds^2. All dimensions must be whole numbers.

Vertical Length (x axis)	Horizontal Length (y axis)	Total Area (yds^2)

How do you know when you have all possible whole-number dimensions?

Is it possible to have non-whole number vales for the dimensions of the rectangle? Explain.

Describe the graph.

Write a rule to describe the graph.

Does the graph touch the “x” axis? Explain.

Does the graph touch the “y” axis? Explain.