Title: Designing a Candy Container: Which Holds Less?	**Grade:** 8	**BIG Idea:** Volume (Secondary concentration on exponents)

CCGPS Standards Addressed:	**Learning Goals:**
MCC8.G.9 Know the formulas for the volume of cones, cylinders, and spheres and use them to solve real-world and mathematical problems. **Mathematical Practices Emphasized:** 1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively. 3. Construct viable arguments and critique the reasoning of others. 4. Model with mathematics 6. Attend to precision	1. Students will calculate the volume of two different cylinders. 2. Students will understand the effect of squaring the radii when comparing the resulting volumes of two different cylinders. 3. Students will determine which cylindrical container will make the most profit for a company.

Materials:

For Teacher:

2 pre-made cylinders from 2 sheets of 8 ½" x 11" paper (cardstock preferable)
Pictures of candy packaged in cylindrical containers (search Google Images)
Video links – see below in "Teacher Notes"

Per Student Pairs:

2 sheets of 8 ½" x 11" paper (cardstock preferable – can be used with multiple classes)
Tape to fasten cylinders
Calculator

Teacher Notes:

Lead a discussion about the economic downturn and how it has affected the sizes of containers. The first link is a short video about food, personal care, and cleaning items whose packages have shrunk, but the costs have not. The second link is still images of items with written commentary.

http://abcnews.go.com/blogs/lifestyle/2012/09/shrinking-products-paying-the-same-for-less/

http://www.ocregister.com/articles/smaller-341845-price-mars.html?pic=2

Students should understand that reducing container size and, therefore, what the container will hold helps companies cut costs while they maintain the same selling prices.

Present the following scenario to students (or a similar one created by the teacher):

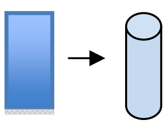
At Christmas (or other appropriate time depending on when task is used), many people buy candy to give as presents. The Jelly Bean Company wants to design a new cylindrical container that looks good but doesn't hold too much candy. However, they want to make customers think they are getting a good deal. As a design team, you will create the candy container that will help the company maximize its profits. (See example below).

Tell students that their mock containers will have no top or bottom, i.e., open cylinders. The raw material for making the containers is cardstock. Tell students to make two different cylinders using the 8 ½" x 11" sheets. <u>DO NOT</u> overlap the paper as this will change the volume calculations.

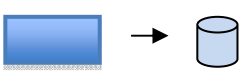
Lead a discussion where students predict which cylinder will hold less and why. Recall that the candy company will choose the container that holds less to increase profit. Ask for justification of guesses. Ask students why the volume would be different since the same sized raw materials (i.e., sheet of cardstock) are being used to make both cylinders.

Determine a way to differentiate between the two cylinders. An example is shown below.

Cylinder 1
Paper orientation - portrait
"taller and slimmer"



Cylinder 2
Paper orientation - landscape
"shorter and wider"



Ask students what measurement needs to be calculated in order to determine how much the container will hold. Volume is the amount of space occupied by an object. The amount of space occupied by the cylinders is their volume. Ask students how the volume of a cylinder can be determined without offering the formula. Have them analyze the cylinders they have made. Remind students of the dimensions of both sheets of paper, 8 ½" x 11".

Students should reason that a cylinder is, basically, a stack of circles. Ask students how this information can help them determine a volume formula.

Area of the base is πr^2 multiplied by the number of circles, or the **height**

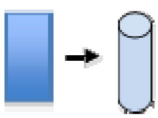
Volume = $\pi r^2 h$

Ask students what information they will have to find out (from the cylinders) in order to determine the volume.

- To find the radius, the diameter must be found. To find the diameter, use the formula for circumference, or $C = \pi d$. Students must solve for diameter given a known circumference and the approximate value of pi. The circumference is the distance around the base of a 3-D cylinder, or the horizontal dimension of a 2-D piece of cardstock.

- Determine the height given the orientation of the paper when forming the cylinder. Below are the calculations for the cylinders 1 and 2.

Cylinder 1 -- taller and slimmer



$C= \pi d$

8.5” = 3.14(d)
2.71” ≈ d

$\frac{2.71}{2} = r$

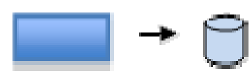
1.4” ≈ r

Volume = $\pi r^2 h$

V = (3.14)(1.4”)^2 (11”)

V ≈ 67.7 in^3

Cylinder 2 – shorter and wider



$C= \pi d$

11” = 3.14(d)
3.5” ≈ d

$\frac{3.5}{2} = r$

1.8” ≈ r

Volume = $\pi r^2 h$

V = (3.14)(1.8”)^2(8.5”)

V ≈ 86.5 in^3

Cylinder 1 has a lower volume than **Cylinder 2**. The candy company should choose **Cylinder 1** in order to make more profit by filling the containers with less candy. Ask students what, mathematically, causes Cylinder 1 to have a lower volume. Have them examine both volume calculations.

V = (3.14)(1.4”)^2 (11”)

V = (3.14)(1.8”)^2(8.5”)

Pi is a constant in both calculations. The difference between the heights, 11" vs. 8.5" isn't that great. However, have students examine the value the gets squared, the radius. From previous study about exponents, students should recall the rapidity with which a number grows due to squaring it. While the difference between a radius of 1.4" and 1.8" isn't that large, there is a large difference between the values when they are squared.

$1.4^2 = 1.96$
$1.8^2 = 3.24$

<u>EXTENSION</u>: To emphasize the effect on volume of squaring the radii versus the height differences, introduce the following to students:

Given a cylinder with…

Radius – 3"
Height – 10"
Volume ≈ 283 in^3

Version A: increase the **radius** by 1" while maintaining the height

Radius – 4"
Height – 10"
Volume ≈ 502 in^3

Version B: increase the **height** by 1" while maintaining the radius

Radius - 3"
Height – 11"
Volume ≈ 311 in^3

Have students compare 283 in^3 to 502 in^3 and 311 in^3.

- 502 in^3 is over 75% larger than 283 in^3

- 311 in^3 is only about 10% larger than 283 in^3