emcsquared: Creating a Tiled Garden

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Title:	**Grade**: 6	**BIG Idea**:
Creating a Tiled Garden *Based on NCTM's *Navigating through Algebra in Grades 6-8, p. 10-11*	**Authors**: Scewilla Elliott Pearlie Jenkins Hope Phillips Mary Lou Wilson	Writing Rules to Identify Algebraic Patterns

Prior Knowledge Needed:

- How to use an input-output table
- What a variable is
- How to plot points on a coordinate plane

GPS Standards:

M6A2: Students will consider relationships between varying quantities.

a. Analyze and describe patterns arising from mathematical rules, tables, and graphs.

Process Standards:

M6P1. Students will solve problems using appropriate technology.

M6P3. Students will communicate mathematically.

M6P5. Students will represent mathematics in multiple ways.

Objectives:

1. Students will understand the relationship between two variables.

2. Students will use words, pictures, tables, and graphs to represent the relationship between two variables.

3. Students will create recursive and explicit rules to describe a figural pattern.

4. Students will determine whether a number is a solution to an algebraic equation.

5. Students will graph the relationship between two variables.

Materials:

Virtual square tiles (see *Resources* section below)
Smart Board (if available)
Online Graphing Calculator - (see *Resources* section below)
Square tiles - about 20 per student or student pair
Handout (see *Resources* section below)
Laptops for each student or a computer lab (if available)
Digital projector

Task:

Given the pattern below, students will answer the following questions:

1. How many tiles are needed to Garden 5? How do you know? Draw Garden 5.
2. Would you like to draw the arrangement for Garden 50? Why or why not?
3. How could you quickly determine the number of tiles needed to make Garden 50? Explain. Be sure to include the total number of tiles.
4. Describe in words how the number of tiles changes from garden to garden.
5. What is the (recursive) rule for the next garden if you know the previous garden? Use words and symbols.
6. What is the rule for the next garden if you know the previous garden? Use words to explain.
7. What is the rule for find the n^{th} garden? Use symbols to explain.

Description and Teacher Directions:

This activity helps students discover both figural and numerical patterns using a variety of mathematical representations (i.e. verbal; tabular; graphical; and symbolic).

Display the handout (see *Resources* section) and tell students they will be creating "gardens" with virtual square tiles. If multiple computers are not available, the teacher may display the virtual manipulatives for the entire class. Students should re-create the tile patterns using virtual manipulatives or square tiles provided by the teacher. The teacher will guide the class allowing individuals/pairs to work independently to determine the answers to the questions above. Students should be in continual dialogue with each other, the teacher, and the class as discoveries are made.
The pattern may be displayed (using digital projector and computer) for the class or passed out as a handout.

Ask students to re-create Gardens 1-4 using tiles (virtual or actual). This re-creation can foster a sense of "how each new garden is made" based on the previous garden, or a recursive method of finding the new garden. Ask students if they can describe in words what is happening to each new garden. Focus on what is recurring (happening over and over) so that a discussion of patterns may emerge. Make sure students can successfully create Garden 5 (question #1) before proceeding to questions #2 - #7.

Teacher Commentary:

My students stayed engaged. Using the manipulatives and the online graphing calculator was a great idea!

My least favorite thing was the students didn't get a chance to input the data into the calculator themselves. If I were teaching this lesson, I would just add laptops.

I liked the way that the teacher used the SmartBoard and then had the students do the same with their manipulatives. Using manipulatives on the SmartBoard is an effective teaching technique.

The teacher can scribe for students who share, or students can write their own explanations for the class. Students should select their own symbols to represent the variables, *Garden number* and *Total Number of Tiles*. Consider allowing students to choose something meaningful to them, instead of selecting for them. Before graphing this relationship, the variables can be likened to "x" and "y" if students chose different variables.

Multiple responses are acceptable. If "n" is the Garden number, below are some of the patterns students might notice in determining the total number of tiles. These are not the only patterns, however.

The following explanations are based on NCTM's N*avigating through Algebra Grades 6-8* and an NCTM E-Workshop:

- **3 + n groups of 2 + 3** [The 3s refer to each column of three tiles on the left and right sides; *n groups of 2* refers to the pairs of two on the top and bottom rows; the number of pairs is equal to the Garden number]

  3 + n x 2 + 3

- **3 + 2 groups of n + 3** [The 3s refer to each column of three tiles on the left and right; *2 groups of n* refer to the number of tiles on the top and bottom rows that are *in between* the two columns of 3 tiles each; the number of tiles in the top and bottom rows, not including the two columns of 3 tiles each, is equal to the Garden number]

  3 + 2 x n + 3

- **2 on the sides + 2 groups of (n + 2)** [The *2 on the sides* refers to the middle tiles in the columns on the left and right; the *2 groups of (n + 2)* refer to tiles in the top and bottom rows; *n + 2* is the sum of the tiles on the top and bottom rows that are the same as the Garden number plus the corner two tiles

  2 + 2 x (n + 2)

Encourage as many explanations as possible. Make sure to record each explanation in words. Later in the lesson, students should go back and try to match a symbolic representation to each written description.

Ask students whether they would like to create Garden 50. Presumably, they would *not* because it would take too many tiles and too much time. Ask them if there is a way to find out the number of tiles for Garden 50 without actually creating it. Remind students that sometimes a tabular representation can help one discern whether a pattern exists.

Create a table like the one below.

Garden Number (note which letter will represent this variable)	**Total Number of Tiles** (note which letter will represent this variable)
1	8
2	10
3	12
4	14
5	16

From the table (above) students should see that two tiles are being added to a previous garden to get the next garden. Here we have a **recursive rule**, a rule that allows us to find the current total based on the previous total.

For Garden 50, it would take a long time to extend the table because one would need to know the total number of tiles in Garden 49 to be able to determine the total number of tiles for Garden 50.

Total Tiles for Garden 50 = Total Tiles for Garden 49 + 2 New Tiles

Some students might suggest the following plan for finding the total tiles for Garden 50:

50-5 = 45

45 is the number of gardens between Garden #5 where we are currently and where we want to be, Garden #50. If we multiply 45 gardens by 2...

45 x 2 = 90 we find out that 90 tiles must be added. However, Garden 5 already has 16 tiles to which we must add the 90. So,

16 + 90 = 106

Ask students if they can think of a rule to represent symbolically the total number of tiles for *any* garden. This type of rule is **explicit** because it stands alone and doesn't require one to know the number of tiles in the previous garden. It will work for any Garden number. Help students think of it as a "one-size-fits- all" rule.

Encourage students to think about what is changing between each garden and what remains constant. Because there are multiple ways to represent the rule symbolically, display as many of the students' "rules" as possible. If possible, match these "rules" to the verbal descriptions recorded on the board from earlier in the lesson. Ultimately, every version of the rule should simplify to the following:

Total number of tiles = 6 + 2(Garden Number)

or

t = 6 + 2g

or

y = 6 + 2x

Discuss with students the value of an **explicit rule** versus a **recursive rule.** If I know an explicit rule, I can find the total number of tiles for *any* garden. Ask students the number of tiles for the Garden 1000. Such a large garden number will help them see the value of the explicit rule.

Using the free, online graphing calculator, graph the Total Number of Tiles (y value) versus the Garden Number (x value). At first, enter the data as individual ordered pairs using the "Plot Points" option. In order to view all of the points, use the "Zoom In" or "Zoom Out" option.

Discuss the linear nature of the graph. Discuss with students, informally, that lines suggest constant change. What change did they observe in the figural and numerical patterns? The addition of two tiles for each new garden. This *rate of change*, or how the pattern changes, is constant.

Although the graph is linear, connecting these points would suggest that the data are continuous. Rather, the data are discrete because there cannot be a partial Garden. For example, there is no Garden1 1/2 ; or 2 3/4;.

This graphing utility does not allow both points and a line to be graphed simultaneously. Enter the explicit rule in the "Y_1=" line. At this point, both variables must be converted to *x* and *y* because that is all the utility recognizes. By clicking "Graph", the individual data points will be replaced by a line. A table of values appears. Entering a value in the "X" column will automatically change the value in the "Y" column.

Discuss the relationship between the two variables. As the Garden Number increases, the total number of tiles increases, as well.

Resources:

http://www.nlvm.usu.edu
National Library of Virtual Manipulatives site URL
for pattern blocks click on Grades 3-5; Geometry; Pattern Blocks

or

http://nlvm.usu.edu/en/nav/frames_asid_169_g_1_t_2.html?open=activities
Direct link to pattern blocks

http://my.hrw.com/math06_07/nsmedia/tools/Graph_Calculator/graphCalc.html
Link to free, online graphing calculator

Student Handout
tiling a garden.doc

tiling a garden [answers].doc
Student Handout Answers

Assessment Item
Assessment [Tiling].doc

Assessment Items Answers