Title:	Grade: 6	BIG Idea:
Chocolate Knows Your Age: The Magic of Math		Using variables to represent numbers; writing expressions

CCGPS Standards Addressed:	Learning Goals:
MCC6.EE.2 Write, read, and evaluate expressions in which letters stand for numbers. MCC6.EE.2c Evaluate expressions at specific values of their variables. Include expressions that arise from formulas used in real-world problems. Perform arithmetic operations, including those involving whole-number exponents, in the conventional order when there are no parentheses to specify a particular order (Order of Operations). **MCC6.EE.6** Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set. **Standards for Mathematical Practice:** 1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively. 4. Model with mathematics. 7. Look for and make use of structure.	1. Given a riddle students will determine the underlying mathematics. 2. Students will create a mathematical expression to represent a real-world problem. 3. Students will evaluate the expression they create.

Materials:
For Teacher: Copy of riddle projected on board

Per Student or Student Pairs: Paper/pencil

Teacher Notes:

The internet is full of riddles and puzzles, and, often, one might receive these via email. Friends find them amusing and pass them along. Engage students in the fun of this silly riddle and, then, ask them why it works.

The mathematical riddle below is entitled, **"Chocolate Knows Your Age."**

1. Pick the number of times a week that you would like to have chocolate -- more than once but less than 10 (e.g., $1<c<10$)

2. Multiply this number by 2.

3. Add 5.

4. Multiply it by 50.

5. If you have already had your birthday this year, add 1764. If you haven't, add 1763.

(NOTE: The addition of either 1764 or 1763 is based on the year 2014. See table on page 4 to determine what value to add for other years.)

6. Subtract the four-digit year that you were born.

7. You should now have a three-digit number. The first digit is your original number, which is the number of times you want to have chocolate each week. The next two numbers are your age.

Students will enjoy the fact that the riddle "tells" them their ages and the number of times they like to eat chocolate in a week. Ask students why the riddle works? Ask them if they think they can uncover, mathematically, the magic? Talk to students about the power of algebra that will help them turn this riddle into a mathematical expression that will work anytime. By knowing about the numbers that underlie the words, students will have the power to "unlock" the mathematical "magic" of these types of problems. Using algebra in this way is what is meant by the Standard for Mathematical Practice, "Modeling with Mathematics." That is, taking a real-world scenario and looking at the math behind it.

Let's look at an example. If a student is born in 2002, has already had his birthday, and selects 9 times to have chocolate…

9 x 2 = 18
18 + 5 = 23
23 x 50 = 1150

1150 + 1764 = 2914
2914 – 2002 = 912

Note that the hundreds digit is the number of times the student wants chocolate and the number formed by the tens and units digit is the student's age. The student wants chocolate 9 times and is 12 years old.

Let's look at another example. If a student is born in 2002, has *not* had his birthday, and selects 9 times to have chocolate…

9 x 2 = 18
18 + 5 = 23
23 x 50 = 1150
1150 + 1763 = 2913
2913 – 2002 = 911

Note that the hundreds digit is still 9 (for chocolate 9 times), but now the number formed by the tens and units digit is 11 indicating that the student is 11 years old.

Tell students they will examine each direction and turn the words into numerical expressions. Below is a step-by-step explanation.

Direction #1: Pick the number of times a week that you would like to have chocolate -- more than once but less than 10

Ask students what this direction asks them to do (see above). Ask, "Since it is unlikely that everyone in the class will select the same number of times, how can we identify the number of times so that we do not have to write separate expressions for every amount students choose?" Help students realize the need to use a ***variable*** to represent the *number of times.* The use of a variable would work for any number of times, 1 through 9. Remind students that a variable is a letter or symbol used to represent a number. Because the value of the variable can vary, every student can build the same expression and have it represent their personal choice for the number of times they want chocolate in a week.

For this discussion the letter **"c"** will represent the variable.

Also use this direction as an opportunity to discuss inequalities, specifically, compound inequalities. Since the value of the variable, "*c,*" must be more than 1 but less than 10, we have *two* stipulations.

$c > 1$ and $c < 10$

$1 < c < 10$ This **compound inequality** expresses both $c > 1$ and $c < 10$ in a single inequality.

Direction #2: Multiply this number by 2

Given our variable, "*c,*" we must multiply it by 2.

We express that as 2*c*.

Direction #3: Add 5

So far, we have the expression 2c. Now we add 5.

$2c + 5$

Direction #4: Multiply it by 50.

The reference to "it" in "multiple *it* by 50" is the expression or quantity, $2c + 5$.

Help students see $2c + 5$ as a number that is temporarily "disguised" by the use of a variable. Ensure students understand that the *quantity* $2c + 5$ must be multiplied by 50 and needs to be enclosed in parentheses. This is different than the multiplying by 50 at the end of this expression.

Right $\longrightarrow$ $50(2c + 5)$ The equivalent expression is $100c + 250$

Wrong $\longrightarrow$ $2c + 5 \bullet 50$ The equivalent expression is $2c + 250$

Direction #5: If you have already had your birthday this year, add 1764.
If you haven't, add 1763.

Already had birthday $\longrightarrow$ $50(2c + 5) + 1764$

Haven't had birthday $\longrightarrow$ $50(2c + 5) + 1763$

*Note: Adding either 1764 or 1763 is based on the calendar year ***2014***. These values will have to be changed to accommodate a different calendar year. See examples below.

Year	Have had birthday	Have not had birthday
2012	1762	1761
2013	1763	1762
2014	1764	1763
2015	1765	1764
2016	1766	1765
	and so on…	

Direction #6: Subtract the four-digit year that you were born.

(Note: The values *1764* and *1763* are based on the year 2014.)

In the expressions below, "y" is one's birth year.

Already had birthday ⟶ 50(2c + 5) + 1764 - y

Haven't had birthday ⟶ 50(2c + 5) + 1763 - y

Directions #7: You should now have a three-digit number.

The first digit is your original number, which is the number of times you want to have chocolate each week. The number formed by the tens and units digit is your age.

Suppose the number of times you wanted chocolate was 9, you were born in 2002, and you have *already had* your birthday. Below is the mathematics representing the directions in the riddle –

50(2c + 5) + 1764 – y	
100c + 250 + 1764 – y	**Simplify the expression by distributing the 50 over the sum of 2c and 5 and then combining like terms.**
100c + 2014 – y	**Simplify addition first (will allow student to see that the sum of 250 and 1764 is the most recent year he/she had a birthday)**
100c + 2014 - y	**Now subtract (difference between calendar year of most recent birthday and one's year of birth is one's age)**
100(c) + 2014 - y	**Formula for determining answer to riddle in 2014**

The expression 100c + 2014 – y will always yield the "magic" three-digit number (based on the year 2014). Why?

- Multiplying by 100 produces a three-digit number.
- The variable, "c," multiplied by 100 produces the digit in the hundreds place that yields the number of times one wants ice cream.
- The difference between the year of one's most recent birthday and the year one was born equals one's age. This value yields the digits in the tens and ones places.

We can use the formula 100(c) + 2014 - y to verify the accuracy of our earlier work. Recall the facts -- a birth year of 2002 and a desire to have chocolate 9 times.

$100(9) + 2014 – 2002 =$
$900 + 2014 – 2002 =$
912

Extension: This formula will *not* work for every possible age. What is the maximum age to which we can apply the formula?

Because the formula results in a three-digit number, a person's age must be $1 < \text{age} < 99$. If a person were 100 years old or older, a four-digit number would be required, thereby needing an additional place value.