Title: Fish Aquariums	**Grade:** 8 **Author**: Hope Phillips	**BIG Idea:** Linear Functions

Real-World Connection:

Fish aquariums can be found in homes, restaurants, and businesses. From simple gold-fish to exotic species, aquariums fascinate children and adults who have ever wanted to capture a small piece of the sea and its contents for their personal enjoyment. The size of an aquarium tank is identified by the number of gallons of water it will hold. For example, the tank size for a 12” x 6” x 8” vessel is 2 ½ gallons. The dimensions and shape of the tank depend on the amount of space available for the aquarium and the client’s preferences.

How Students will Experience the Connection: highlight in yellow all that apply

Video Clip (**Futures’ Videos -- *Aquarium Makers***) Photo Podcast

Print Media (article, ad, etc.) Vodcast Other

GPS Standards	**Objectives**:
M8A3. Students will understand relations and linear functions. i. Translate among verbal, tabular, graphic, and algebraic representations of functions. **M8A4.** Students will graph and analyze graphs of linear equations and inequalities. a. Interpret slope as a rate of change. b. Determine the meaning of the slope and *y*-intercept in a given situation. c. Graph equations of the form *y* = *mx* + *b*. f. Determine the equation of a line given a graph, numerical information that defines the	. 1. Students will calculate the volume of aquariums with given dimensions. 2. Students will determine a ratio between the *volume* of an aquarium and *tank size* (number of gallons of water it holds). 3. Students will use graphing calculators or an online graphing utility to plot data points and the resulting linear function. 4. Students will interpret the meaning of the slope and y-intercept, generated by the calculator, in the context of the problem.

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line or a context involving a linear relationship. g. Solve problems involving linear relationships. **M8P1.** Students will solve problems (using appropriate technology). **M8P3.** Students will communicate mathematically. **M8P4.** Students will make connections among mathematical ideas and to other disciplines. **M8P5.** Students will represent mathematics in multiple ways.	

Materials:

- Handout – per student
- Graphing calculator – one per student or student pair OR computer lab with online graphing utility (examples include: TI-73; TI-83; TI-83+; TI-84; TI-84+;TI-Inspire; Casio graphing calculators)
- Overhead view screen or emulator if using graphing calculator
- Four-function calculator if using computer lab option – one per student or student pair
- Inch cubes – enough for demonstration by teacher
- Small rectangular aquarium or shoebox to represent rectangular aquarium - optional

Related Task:

The use of a graphing calculator is often a necessity when using real-world problems. The data may not be "friendly" for graphing by hand. The data in this problem includes "x" values ranging from 576 to 43,200. While students do not get the benefit of building the coordinate plane, they must use their data to set up an appropriate viewing window which requires mathematical understanding.

Introduce students to aquariums using the "Aquarium Makers" or a similar online video clip. Engage students in a conversation about where one can buy an aquarium. Many students will have seen them in the pet section of a big-box store. (Note: The lesson that follows is about rectangular aquariums only). Ask them how the retail clerk knows how much water is needed for any size tank. Likely, the clerk refers to a chart comprised of water specifications by tank size. Ask students how they think the pre-made chart

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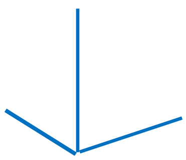
specifications were determined. The mathematical foundation of the lesson lies in this question.

Determine students' prior knowledge about volume of a rectangular prism. Ask them what measurement determines the amount of water that will be contained in an aquarium. Students may confuse area and volume, but remind them that for a 3-D figure like an aquarium, there is the extra dimension of height. Volume is the amount of space an object occupies and includes the materials of which the object is composed. Capacity, more accurately, describes the amount of water an aquarium will hold. However, these terms are often used interchangeably. For this lesson, *volume* will refer to the amount of water the aquarium will hold. Draw something like the following on the board:



This drawing represents a corner of an aquarium and can be used to model the dimensions *length*, *width*, and *height*. Aquariums, then, are measured in cubic inches (in^3 or in. x in. x in.). A visual will help students see the three dimensions and link the cubic nature of the units of measure.

Ask students how volume is calculated. Some students may prefer $V_{rectangular\ prism} = \mathbf{l} \times \mathbf{w} \times \mathbf{h}$. Others may prefer the general formula for a prism $V = Bh$ where "B" is the area of the base ($l \times w$). Volume can be represented by the amount of water that would fill 1 in^3. Show students several inch cubes to help them visualize how aquarium volume is determined. For example, for the 2.5 gallon tank (12" x 6" x 8"), students can picture 576 inch cubes filling the tank and the water contained in each of them, collectively, would be the amount of water in that aquarium.

Pass out the handout and graphing calculators. Ask students to articulate what is involved in finding the ratio of the volume to the tank size. The volume must be calculated using the aquarium dimensions. Focus on what the meaning of *ratio* (comparison of two values by division) and how to key in the values in the calculator. Ask students to predict the value of the ratio – greater than one or less than one. Many students will incorrectly key in the larger number divided by the smaller number because they are accustomed to working with numbers greater than one. Because we are dividing a smaller number by a larger number, the value will be less than one and very small. Students should round their answers to the ten-thousandths place. A review of rounding may be required.

Students may be assigned the finding of ratios by section (small/mid-sized/ and large values), individually, or a way of the teacher's choosing. Students need to report their findings so that all students have all the data. Discuss the nature of the ratio values. Students should note that all of the values are very close to the same number, approximately 0.0042. Ensure that students use the units of this value when talking

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about it -- $\frac{0.0042\ gallons}{in^3}$ Tell students they will be investigating this ratio more after entering their data in the graphing calculators.

Have students enter into their graphing calculators the data from the handout as ordered pairs (*tank size, volume).* Next, determine a viewing "window." Students should examine the data on the handout to determine appropriate window settings. For example, below is a sample window –

Xmin – 550
Xmax – 44000
Xscl – 5000
Ymin- 0
Ymax – 200
Yscl – 50

*****A "rule of thumb" for setting the window is that the settings should allow all data points to be shown on the screen when using the *trace* button.

After the window is set, graph the data. Ask students to describe the graph (i.e. appears linear). If a graph is linear, there must be a constant rate of change. Ask students if they recall a value that was constant or almost constant in the work they have been doing. The ratio of *volume* to *tank size* is a constant, although in students' data it may have varied somewhat.

Ask students to look at the units of the ratio, as well as the approximate rounded value of 0.0042. It is vital that students include the units with the value of the slope. The units help us interpret the *meaning* of the slope in the context of the problem. The ratio

$\frac{0.0042\ gallons}{in^3}$ refers to the amount of water contained in a cubic inch of space. There are 0.0042 gallons of water for every cubic inch of space.
No matter the size of the tank, there is a *constant* amount of water per cubic inch of space.

Obviously, it is very difficult for students to envision this small amount of water. One option is to refer to the inch cubes and tell students that the amount of water that would be contained in that cube is 0.0042 gallons. The other option is to simplify the reciprocal of this value. $\frac{1\ in^3}{0.0042\ gallons} \approx \frac{238\ in^3}{gallon}$ In this form, the ratio may have more meaning to students. There are approximately 238 in^3 of volume in every gallon of water.

Ask students how knowing this ratio could be helpful. Using $\frac{238\ in^3}{gallon}$, students could estimate the size of the tank based on the volume. For example, if students look at the data on their handouts, the first aquarium has a 2.5 gallon tank with a volume of 576 $in^{3.}$

If one didn't know the tank size, it could be found based on our ratio and the given volume of 576 $in^{3.}$ See work below.

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576 in^3 $\div \frac{238\ in^3}{gallon} \approx$ 2.4 gallons   2.4 this very close to the actual number of gallons, 2.5

If one didn't know the volume but knew the tank size, the volume could be found using our ratio to find the volume. See work below.

$\frac{238\ in^3}{gallon}$ x 2.5 gallons = 595 in^3

According to the data on the handout, the actual volume for a 2.5 gallon tank is 576 in^3. However, remind students that we estimated the slope at $\frac{0.0042\ gallons}{in^3}$, so our answer, then, is also an estimate.

Use the linear regression function of the calculator to estimate a line-of-best fit. The equation is y = 0.0042x + 0.27. Students will recognize the value of the slope, but they may struggle with understand the meaning of the y-intercept. Theoretically, if there is no tank, there is no volume. Students might have anticipated the equation to have been y = 00.42x. However, because the amount of water per cubic inch is an estimated value, there is a small amount of error, hence a y-intercept value of 0.27. This value cannot be interpreted in the context of the problem. If there is no tank, there would not be a volume of 0.27 in^3. The important point for students is that real-world data does not always conform to rules and produce "friendly" values.

So that students can use the equation they have generated, ask them to find the tank size if they have the following:

aquarium dimensions $\longrightarrow$ 96" x 48" x 60"

Volume = 96" x 48" x 60"
= 276,480 in^3

y = 0.0042x + 0.27
y = 0.0042 (276,480) + 0.27
y = 1161.2 gallons

Pose the following problem: The total amount of water in the Georgia Aquarium tanks is 8,000,000 gallons. What is the total volume of all of the aquariums?

y = 0.0042x + 0.27
8,000,000 = 0.0042x + 0.27
1,904,761,905 in^3 = x

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Learn More:

For general information about aquariums –

http://freshaquarium.about.com/od/aquariumsandstands/a/tanksizesweights.htm

*Online graphing utility options:

http://my.hrw.com/math06_07/nsmedia/tools/Graph_Calculator/graphCalc.html

http://www.shodor.org/interactivate/activities/Graphit/

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Directions: Determine the ratio of the tank size to the volume. Round each value to the ten-thousandths place.

Note: Weights are for glass aquariums. Gallons refer to water. Source: http://freshaquarium.about.com/od/aquariumsandstands/a/tanksizesweights.htm

SMALL AQUARIUMS		
Tank Size	**L x W x H**	**Ratio of Tank Size/Volume**
2 1/2 gallon	12" x 6" x 8"	
5 gallon	16" x 8" x 10"	
10 gallon "Leader"	20" x 10" x 12"	
15 gallon	24" x 12" x 12"	
MID-SIZED AQUARIUMS		
Tank Size	**L x W x H**	**Ratio of Tank Size/Volume**
20 gallon High	24" x 12" x 16"	
25 gallon	24" x 12" x 20"	
29 gallon	30" x 12" x 18"	
30 gallon Breeder	36" x 18" x 12"	
40 gallon Long	48" x 12" x 16"	
LARGE AQUARIUMS		
Tank Size	**L x W x H**	**Ratio of Tank Size/Volume**
50 gallon	36" x 18" x 19"	
55 gallon	48" x 13" x 21"	
65 gallon	36" x 18" x 24"	
75 gallon	48" x 18" x 21"	
90 gallon	48" x 18" x 24"	
125 gallon	72" x 18" x 21"	
150 gallon	72" x 18" x 28"	
180 Gallon	72" x 24" x 25"	

Write the equation of the line.

What is value of the slope?

What is the real-world meaning of the slope?

Calculate the tank size for a 96” x 48” x 60” aquarium.